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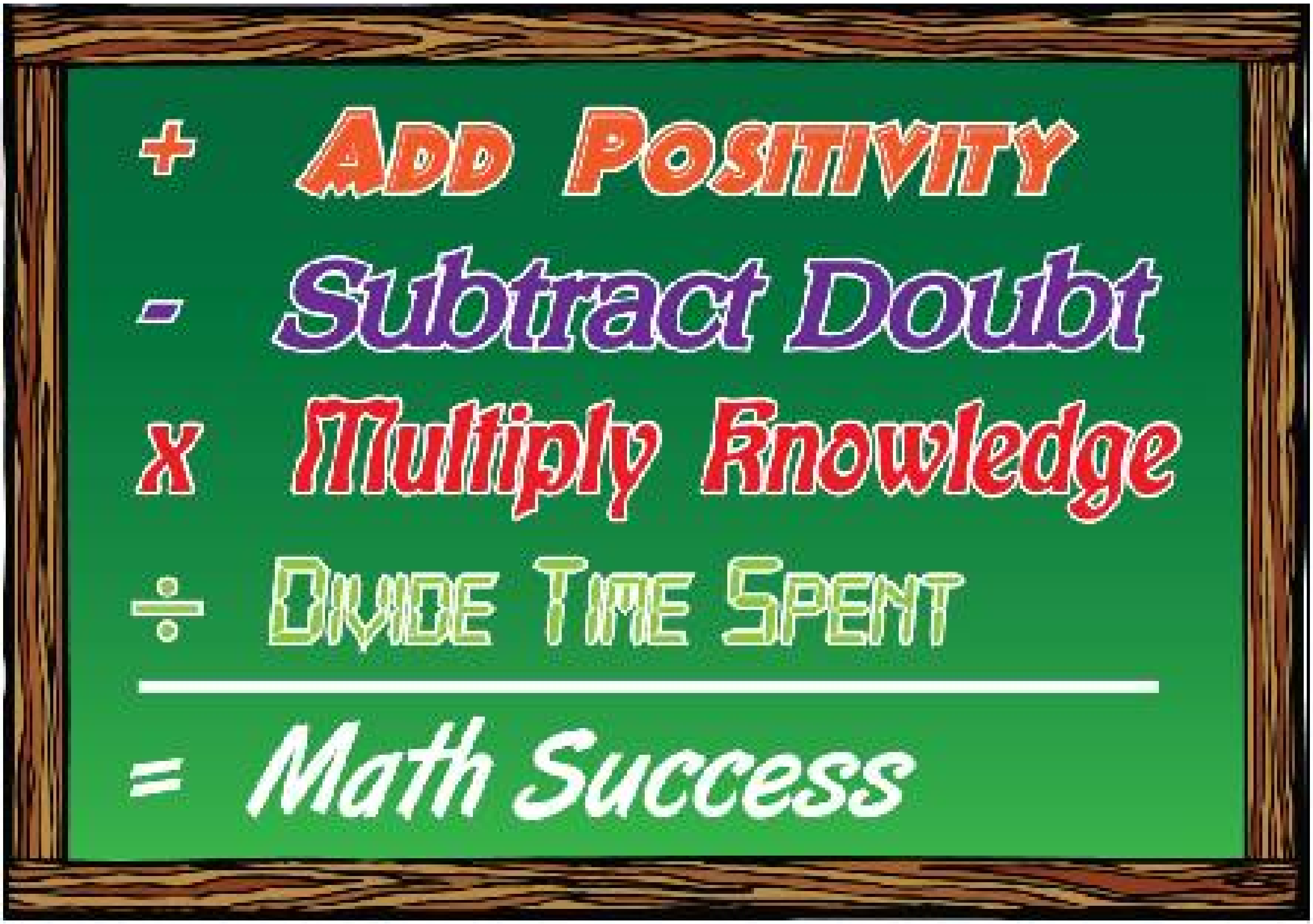

MathCelebrity Study Guide

MathCelebrity

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Contents

1 Basic Math

1.1 Numerical Properties

Property	Description
Reflexive Property	$A \Leftrightarrow A$
Symmetric Property	$If A = B \Rightarrow B = A$
Transitive Property	$If A = B$ and $B = C \Rightarrow A = C$
Zero Multiplication Property	$A * 0 = 0$
Zero Property Additive Identity	$A + 0 = 0$
Multiplicative Identity	$A * 1 = A$
Additive Inverse	$A + (-A) = 0$
Multiplicative Inverse	$A * \frac{1}{A} = 1$
Trichotomy Property	$a < b, a > b, or a = b$
Addition Equality	$If a = b \Rightarrow a + c = b + c$
Subtraction Equality	$If a = b \Rightarrow a - c = b - c$
Multiplication Equality	$If a = b \Rightarrow a * c = b * c$
Division Equality	$If a = b and c \neq 0 \Rightarrow \frac{a}{c} = \frac{b}{c}$
Addition Inequality	$\forall a, b, c \quad \in \mathbb{R} If a < b \Rightarrow a + c < b + c$
Subtraction Inequality	$\forall a, b, c \quad \in \mathbb{R} If a < b \Rightarrow a - c < b - c$
Multiplication Inequality	$\forall a, b, c \quad \in \mathbb{R} If a < b \Rightarrow a * c < b * c$
Division Inequality	$\forall a, b, c \quad \in \mathbb{R} If a < b and c > 0 \Rightarrow a \div c < b \div c$
Associative Property of Addition	$\forall a, b, c \quad \in \mathbb{R}(a + b) + c = a + (b + c)$
Associative Property of Multiplication	$\forall a, b, c \quad \in \mathbb{R}(a * b) * c = a * (b * c)$
Commutative Property of Addition	$\forall a, b, c \quad \in \mathbb{R} a + b + c = c + b + a$
Commutative Property of Multiplication	$\forall a, b, c \quad \in \mathbb{R} a * b * c = c * b * a$
Distributive Property	$\forall a, b, c \quad \in \mathbb{R} a(b + c) = a(b) + a(c)$
Fraction Cancellation	$\forall a, b, c \quad \in \mathbb{R} b \neq 0, and c \neq 0, \frac{a*c}{b*c} = \frac{a}{b}$

1.2 Basic Math Operations

Addition of 2 numbers

Subtraction of 2 numbers

Multiplication of 2 numbers

Division of 2 numbers

Opposite Numbers

Addition of 3 or more numbers

Multiplication Array

4 rows x 3 columns = 12

$$\begin{vmatrix} * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{vmatrix}$$

Multiplication Tables

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

1.3 Estimating Sums - Common Core

1. Round each number
2. Add up the rounded numbers

Example: $440 + 35$

1. Round $440 \Rightarrow 400$
2. Round $35 \Rightarrow 30$
3. $400 + 30 = 430$

1.4 Estimating Reasonableness of Product - Common Core

1. Round each number down
2. Multiply the rounded numbers (a)
3. Take the residual from the first number (Original Number - Rounded Number)
4. Multiply the residual of the first number by the rounded second number (b)
5. Take the residual from the second number (Original Number - Rounded Number)
6. Multiply the residual of the second number by the rounded first number (c)
7. Add up $a + b + c$
8. If the sum is less than the initial estimate, this seems like a valid estimate

Example: $77 * 43$ with an estimated product of 3,311. Using the steps above, we have:

1. 70 and 40

2. $70 * 40 = 2,800$ (a)
3. $77 - 70 = 7$
4. $7 * 40 = 280$ (b)
5. $43 - 40 = 3$
6. $3 * 70 = 210$ (c)
7. $2,800 + 280 + 210 = 3,290$
8. Since $3,290 < 3,311$, this is a reasonable estimate of the product

1.5 Number Bonds

The relationship of a number to the combination of its' parts. We use the number 10 as a marker.

1. Add $8 + 6$ using number bonds of 10
2. Since 6 = 2 + 4, we can write 8 + 6 as 8 + 2 + 4
3. Group by 10
4. (8 + 2) + 4
5. 10 + 4
6. 14

1.6 Lattice Math Multiplication

$14 * 56$

1. Write first number across the top and second number down the sides

1	4	
		5
		6

1. Multiply 1 times 5 to get 5 and split the digits in the upper and lower corners
2. Multiply 4 times 5 to get 20 and split the digits in the upper and lower corners
3. Multiply 1 times 6 to get 6 and split the digits in the upper and lower corners
4. Multiply 4 times 6 to get 24 and split the digits in the upper and lower corners

1	4	
0/5	2/0	5
0/6	2/4	6

Add the diagonal totals up: Start at the bottom right and move to the top left:

1. 4 = 4
2. 0 + 6 + 2 = 8
3. 5 + 2 + 0 = 7
4. 0 = 0

Putting together our answer by reading down from the left upper entry to the bottom right, we get 0-7-8-4 = 784

1.7 Absolute Value

The absolute value of a number is the distance from a point to the origin on the number line. Anything inside the vertical braces becomes positive.

$|a| = a$

$|-a| = a$

$-|-a| = -a$

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & x < 0 \end{cases}$$

1.8 Opposite Numbers

The opposite of $a = -a$

The opposite of $-a = -(-a) = a$

1.9 Midpoint of a Line

Midpoint $\overline{AB} = \frac{a+b}{2}$

1.10 Number Types

Whole - All positive integers including 0 $0, 1, 2, 3, ...$

Natural - All positive integers after 0 $1, 2, 3, 4, ...$

Even - Numbers that can be divided by 2 $\{2, 4, 6, 8, ...\}$

Odd - Numbers not divisible by 2 $\{3, 5, 7, 9, ...\}$

Prime - Numbers with no other factors than 1 and itself $\{3, 5, 7, 11, ...\}$

Composite - Numbers with factors other than one and itself $\{4, 6, 8, 10, ...\}$

Fibonacci Sequence

$$F_n = \begin{cases} 0, & \text{if } n = 0 \\ 1, & \text{if } n = 1 \\ F_{n-1} + F_{n-2}, & \text{if } n > 1 \end{cases} \tag{1}$$

The first ten Fibonacci numbers are $0, 1, 1, 2, 3, 5, 8, 13, 21, 34$

Algorithm 1.10.1 Fibonacci Algorithm

procedure FIBONACCI(n)
    $f_0 := 0$
    $f_1 := 1$
    $counter := 2$
    while $counter \leq n$ **do**
        $f_n := f_{n-1} + f_{n-2}$
        $counter = counter + 1$
    end while
    return f_n ▷ Return the n^{th} Fibonacci number
end procedure

1.11 Comparing Numbers

Comparing Numbers: Given 2 numbers a and b then 1 of the following 3 is true

If $a < b \Rightarrow a - b < 0$

If $a > b \Rightarrow a - b > 0$

If $a = b \Rightarrow a - b = 0$

1.12 Number Operations

Round Truncate Ceiling Floor: Take a number 95.46783

Operation	Explanation	Example
Round	Round a number to a certain amount of digits	$Round(95.46783, 2) = 95.47$
Truncate	Remove all decimals	$Truncate(95.46783) = 95$
Ceiling	Next highest integer	$[95.46783] = 96$
Floor	Next lowest integer	$]95.46783[= 95$
Accuracy	Number of digits to the right of the decimal point	95.46783 has an accuracy of 5
Precision	Number of digits in the number	95.46783 has an precision of 6

Floor of x is also written as $\lfloor x \rfloor$

Ceiling of x is also written as $\lceil x \rceil$

1.13 Tally Marks

Every time you reach a multiple of 5, $(5, 10, 15, 20)$, you use a horizontal line through the tally marks

Number	Tally Marks
1	|
2	||
3	|||
4	||||
5	𝍸
6	𝍸 |
7	𝍸 ||
8	𝍸 |||
9	𝍸 ||||
10	𝍸 𝍸
11	𝍸 𝍸 |
12	𝍸 𝍸 ||
13	𝍸 𝍸 |||
14	𝍸 𝍸 ||||
15	𝍸 𝍸 𝍸
16	𝍸 𝍸 |
17	𝍸 𝍸 ||
18	𝍸 𝍸 |||
19	𝍸 𝍸 ||||
20	𝍸 𝍸 𝍸 𝍸

1.14 Count Backwards

Count Backwards

n

$n - x$

$n - 2x$

$n - (n-1)x$

1.15 True False Equations

Determine if the left side of the equation equals the right side of the equation.

Determine if $4 + 1 = 5 + 2$ is true or false.

1. Evaluate the left side: $4 + 1 = 5$
2. Evaluate the right side: $5 + 2 = 7$
3. Since $5 \neq 7$, this is False

1.16 Balancing Equations

1.16.1 Balance Addition Equations

Take 4 numbers, n_1, n_2, n_3, n_4

Using addition, we try to balance the equation such that one of the following is true:

$n_1 + n_2 = n_3 + n_4$

$n_1 + n_3 = n_2 + n_4$

$n_1 + n_4 = n_2 + n_3$

Take the numbers $8, 6, 3, 5$

$8 + 6 = 3 + 5 \Rightarrow 14 \neq 7$

$8 + 3 = 6 + 5 \Rightarrow 11 = 11$

$8 + 5 = 6 + 3 \Rightarrow 13 \neq 9$

1.16.2 Balance Subtraction Equations

Take 4 numbers, n_1, n_2, n_3, n_4

Using subtraction, we try to balance the equation such that one of the following is true:

$n_1 - n_2 = n_3 - n_4$

$n_1 - n_3 = n_2 - n_4$

$n_1 - n_4 = n_2 - n_3$

$n_1 - n_3 = n_4 - n_2$

Take the numbers $5, 6, 2, 9$

$5 - 6 = 2 - 9 \Rightarrow -1 \neq -7$

$5 - 2 = 6 - 9 \Rightarrow -3 \neq 3$

$5 - 9 = 6 - 2 \Rightarrow -4 \neq 4$

$5 - 2 = 9 - 6 \Rightarrow 3 = 3$

1.16.3 Balance Multiplication Equations

Take 4 numbers, n_1, n_2, n_3, n_4

Using multiplication, we try to balance the equation such that one of the following is true:

$n_1 * n_2 = n_3 * n_4$

$n_1 * n_3 = n_2 * n_4$

$n_1 * n_4 = n_2 * n_3$

Take the numbers $8, 10, 4, 5$

$8 * 10 = 4 * 5 \Rightarrow 80 \neq 40$

$8 * 4 = 10 * 5 \Rightarrow 32 \neq 50$

$8 * 5 = 10 * 4 \Rightarrow 40 = 40$

1.16.4 Balance Division Equations

Take 4 numbers, n_1, n_2, n_3, n_4

Using subtraction, we try to balance the equation such that one of the following is true:

$\frac{n_1}{n_2} = \frac{n_3}{n_4}$

$\frac{n_1}{n_3} = \frac{n_2}{n_4}$

$\frac{n_1}{n_4} = \frac{n_2}{n_3}$

$\frac{n_1}{n_3} = \frac{n_4}{n_2}$

Take the numbers $20, 5, 40, 10$

$\frac{20}{5} = \frac{40}{10} \Rightarrow 4 = 4$

$\frac{20}{40} = \frac{5}{10} \Rightarrow 0.5 = 0.5$

$\frac{20}{10} = \frac{5}{40} \Rightarrow 2 \neq 0.125$

$\frac{20}{40} = \frac{10}{5} \Rightarrow 0.5 \neq 2$

1.17 Number Pairs

Number pairs are combinations of two numbers which add up to a given number. See the grid below:

Number	Number Pairs
2	$1+1$
3	$1+2, 2+1$
4	$1+3, 2+2, 3+1$
5	$1+4, 4+1, 2+3, 3+2$
6	$1+5, 5+1, 2+4, 4+2, 3+3$
7	$1+6, 6+1, 2+5, 5+2, 3+4, 4+3$
8	$1+7, 7+1, 2+6, 6+2, 3+5, 5+3, 4+4$
9	$1+8, 8+1, 2+7, 7+2, 3+6, 6+3, 4+5, 5+4$
10	$1+9, 9+1, 2+8, 8+2, 3+7, 7+3, 4+6, 6+4, 5+5$

1.18 Decompose Number Pairs

Decomposing a number into pairs is finding all combinations of two numbers that add to our original number. Example: Decompose 5 into number pairs.

$5 = 1 + 4$

$5 = 2 + 3$

$5 = 3 + 2$

$5 = 4 + 1$

1.19 Unknown Numbers

Unknown Numbers work like equations using a question mark to identify a missing number. What you need to do is find the number that makes the equation work:

$8+? = 11 \Rightarrow ? = 3$

$12-? = 7 \Rightarrow ? = 5$

1.20 Ten Frames

Dot representation on a basis of 10 per card:

Ten-frame for 5

·	·
·	·
·	

Ten-frame for 6

·	·
·	·
·	·

1.21 Duplication and Mediation

We use this method to determine the product of two numbers. Take $471 * 35$

1. Divide the left hand column by 2, and if it is odd, go to the next lowest integer.
2. Double Column 2
3. Cross out any right hand column which having a value in the left hand column that is even
4. Stop when the left hand column equals 1
5. Add up all the right hand columns that have a left hand column value which is odd

35	471
$\frac{35}{2} = 17$	$471 * 2 = 942$
$\frac{17}{2} = 8$	$942 * 2 =$ ~~1,884~~
$\frac{8}{2} = 4$	$1,884 * 2 =$ ~~3,768~~
$\frac{4}{2} = 2$	$3,768 * 2 =$ ~~7,536~~
$\frac{2}{2} = 1$	$7,536 * 2 = 15,072$

$471 + 942 + 15,072 = 16,485 \Rightarrow 471 * 35 = 16,485$

2 Geometry

Geometry Calculators

2.1 2-Dimensional Shape Formulas

Shape	Perimeter	Area	Comments
Circle	$2\pi r$	πr^2	r = radius
Square	$4s$	s^2	s = side
Rectangle	$2l + 2w$	$l * w$	l = length and w = width
Parallelogram	$2l + 2w$	$l * w$	l = length and w = width
Equilateral Triangle	$3s$	$\frac{1}{2}sh$	s = side and h = height
Isosceles Triangle	$2s + b$	$\frac{1}{2}sh$	s = side and h = height and b = base
Kite	$2ss + 2ls$	$d_1 d_2$	ss = short side and ls = long side
Trapezoid	$b_1 + b_2 + s_1 + s_2$	$\frac{a(b_1+b_2)}{2}$	b = base
Pentagon	$5s$	$\frac{a*s}{4}$	s = side
Hexagon	$6s$	$2(1+\sqrt{2})s^2$	s = side
Heptagon	$7s$	$\frac{7}{4}s^2 cot(\frac{180\deg}{7})$	s = side
Octagon	$8s$	$2(1+\sqrt{2})s^2$	s = side
Nonagon	$9s$	$\frac{9}{4}s^2 cot(\frac{180\deg}{9})$	s = side
Rhombus	$4s$	$\frac{d_1 d_2}{2}$	s = side, d_1 = Diagonal 1, d_2 = Diagonal 2
Annulus	N/A	$R^2 - r^2$	R = Outer Radius and r = inner radius

2.2 3-Dimensional Shape Formulas

Shape	Surface Area	Lateral Area	Volume	Comments
Cube	$6s^2$	$4s^2$	s^3	s = side
Rectangular Solid	$lw + lh + wh$	$h(2l + 2w)$	lwh	l = length, w = width, and h = height
Cylinder	$\pi r^2 h + 2\pi rh$	$2\pi rh$	$\pi r^2 h$	r = radius and h = height
Pyramid	$b * LA$	$\frac{BasePerimeter*SlantHeight}{2}$	$\frac{bh}{3}$	b = base and h = height
Sphere	$4\pi r^2$	N/A	$\frac{4\pi r^3}{3}$	r = radius and h = height
Cone	$\pi r^2 + \pi rl$	N/A	$\frac{\pi r^2 h}{3}$	r = radius and h = height
Torus	$4\pi^2 Rr$	$4\pi^2 Rr$	$(\pi r^2)(2\pi R)$	r = minor radius and R = major radius

2.3 Cuboid

Cuboid Measurement	Formula
Volume	$a * b * c$
Surface Area	$2(ab + bc + ca)$
Face Diagonal Length d_{ab}	$\sqrt{a^2 + b^2}$
Face Diagonal Length d_{ac}	$\sqrt{a^2 + c^2}$
Face Diagonal Length d_{bc}	$\sqrt{b^2 + c^2}$
Space Diagonal Length d_{abc}	$\sqrt{a^2 + b^2 + c^2}$

2.4 Triangle Items

Angle Ratio for a Triangle $\Rightarrow ax + bx + cx = 180$

Cevian Triangle Items $\Rightarrow a^2n + b^2m = t^2c + mnc$

Centroid $= (\frac{x_1+x_2+x_3}{3}), (\frac{y_1+y_2+y_3}{3})$

Geometric Mean of a Triangle

$\frac{\overline{AD}}{\overline{CD}} = \frac{\overline{CD}}{\overline{DB}}$

Triangle Inequality

$\triangle ABC \Rightarrow \overline{AB} + \overline{AC} > \overline{BC}, \overline{AB} + \overline{BC} > \overline{AC}, \overline{AC} + \overline{BC} > \overline{AB}$

2.4.1 Special Triangles

30-60-90 Triangle Side Ratios opposite each angle are $1, \sqrt{3}, 2$

45-45-90 Triangle Side Ratios opposite each angle are $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1$

2.5 Clocks

Clock Angle Calculator

$\theta H = \frac{60H+M}{2}$

$\theta m = 6M$

$\Delta\theta = |\theta h - \theta m|$

Clock Hands Meet

1. For n = 0 to 11 $\frac{12n}{11}$
2. Hour = Integer Portion
3. Minutes = Integer(60 * Decimal Portion)
4. Seconds = 60 * Decimal Portion

2.6 Planar

Eulers Planar Formula$\Rightarrow v - e + f = 2$ where v = vertices, e = edges, and f = faces

Polygons

$P = ns$

$A = \frac{s^2n}{4tan\frac{\pi}{n}}$

Interior Angle Sum $= (n-2) * 180°$

Diagonals $= \frac{n(n-3)}{2}$

Vertex Diagonals $= n - 3$

Triangles from one Vertex $= n - 2$

Polygon Sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexatagon
7	Heptagon
8	Octagon
9	Decagon
10	Dodecagon
11 or more	n-gon

2.7 Quadrilaterals

Quadrilateral Area: $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$

Bretschneiders Formula Area: $A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd \cdot cos^2(0.5(\alpha+\gamma))}$

2.8 3 Dimensional Points

Given two 3 Dimensional Points (x_1, y_1, z_1) and (x_2, y_2, z_2):

Distance between the points $D = \sqrt{(x_2 - y_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Parametric Equations: $(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Symmetric Form Equation: $\frac{x-x_0}{a}, \frac{y-y_0}{b}, \frac{z-z_0}{c}$

2.9 Golden Ratio

The Golden Ratio ϕ states: Large Segment A, Small Segment B, and total segment $\overline{AB} \Rightarrow \frac{A}{B} = \frac{\overline{AB}}{A}$

Identities: $\phi + 1 = \phi^2$ and $\phi - 1 = \frac{1}{\phi}$

3 Trigonometry

TRIGONOMETRY Calculators

3.1 Angles and Angle Measurements

Given an angle θ:

$\theta°$	$\theta^{radians}$	$sin(\theta)$	$cos(\theta)$	$tan(\theta)$	$csc(\theta)$	$sec(\theta)$	$cot(\theta)$
$0°$	0	0	1	0	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$90°$	$\frac{\pi}{2}$	1	0	N/A	1	0	N/A
$120°$	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{-1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$\frac{-\sqrt{3}}{3}$
$135°$	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$-\sqrt{2}$	-1
$150°$	$\frac{5\pi}{6}$	$\frac{1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{3}$	2	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{3}$
$180°$	π	0	-1	0	0	-1	N/A
$210°$	$\frac{7\pi}{6}$	$\frac{-1}{2}$	$\frac{-\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{3}$	-2	$\frac{-2\sqrt{3}}{3}$	$-\sqrt{3}$
$225°$	$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
$240°$	$\frac{4\pi}{3}$	$\frac{-\sqrt{3}}{2}$	$\frac{-1}{2}$	$-\sqrt{3}$	$\frac{-2\sqrt{3}}{3}$	-2	$\frac{-\sqrt{3}}{3}$
$270°$	$\frac{3\pi}{2}$	-1	0	N/A	-1	0	N/A
$300°$	$\frac{5\pi}{3}$	$\frac{-\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$\frac{-2\sqrt{3}}{3}$	2	$\frac{-\sqrt{3}}{3}$
$315°$	$\frac{7\pi}{4}$	$\frac{-\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$\sqrt{2}$	-1
$330°$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{-\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$

3.2 Related Angles

- Complementary Angle (Degrees) $90 - \theta$ where $\theta < 90$
- Complementary Angle (Radians) $\frac{\pi}{2} - \theta$ where $\theta < 90$
- Supplementary Angle (Degrees) $180 - \theta$ where $\theta < 180$
- Supplementary Angle (Radians) $\pi - \theta$ where $\theta < 180$
- Coterminal Angles (Degrees) $\theta \pm 360°$
- Coterminal Angles (Radians) $\theta \pm 2\pi$
- Reference Angle $R(\theta) = 180 - \theta$ where $0 < \theta < 180$
- Reference Angle $R(\theta) = 360 - \theta$ where $180 < \theta < 360$

3.3 Classify Angles

- Acute Angles when $\theta° < 90$
- Right Angles when $\theta° = 90$

- Obtuse Angles when $\theta^\circ > 90$
- Straight Angles (Special Obtuse Angle) when $\theta^\circ = 180$

3.4 Degrees Minutes Seconds

Degrees Minutes Seconds to Decimal Degrees $\Rightarrow d^\circ + \frac{m}{60} + \frac{s}{3600}$

Decimal Degrees to Degrees Minutes Seconds

$D = Int(d^\circ)$

$M = 60 * (Int(d^\circ) - d^\circ)$

$S = 60 * (M - Int(M))$

3.5 Trig Functions

(x, y) is a point other than the origin. Distance from that point to the origin is $r = \sqrt{x^2 + y^2}$

$sin\theta = \frac{y}{r} = \frac{opposite}{hypotenuse}$

$cos\theta = \frac{x}{r} = \frac{adjacent}{hypotenuse}$

$tan\theta = \frac{y}{x} = \frac{opposite}{adjacent}$

$csc\theta = \frac{r}{y} = \frac{1}{sin\theta}$

$sec\theta = \frac{r}{x} = \frac{1}{cos\theta}$

$cot\theta = \frac{x}{y} = \frac{1}{tan\theta}$

3.6 Pythagorean Theorem

Pythagorean Theorem $a^2 + b^2 = c^2$

$sin\theta^2 + cos\theta^2 = 1$

$sec\theta^2 + tan\theta^2 = 1$

$cot\theta^2 + 1 = csc\theta^2$

3.7 Cofunctions

For any acute Angle A, the cofunction is listed below:

$sinA = cos(90^\circ - A)$

$cosA = sin(90^\circ - A)$

$cscA = sec(90^\circ - A)$

$secA = csc(90^\circ - A)$

$tanA = cot(90^\circ - A)$

$cotA = tan(90^\circ - A)$

3.8 Trig Conversions

$Radians = \frac{\theta^\circ \pi}{180}$

$Gradians = \frac{10\theta}{9}$

Angle Ratio for a triangle with angles a, b, and c in the form a:b:c $= ax + bx + cx = 180$

3.9 Negative Angle Identities

$sin(-\theta) = -sin(\theta)$

$cos(-\theta) = cos(\theta)$

$tan(-\theta) = -tan(\theta)$

$csc(-\theta) = -csc(\theta)$

$sec(-\theta) = sec(\theta)$

$cot(-\theta) = -cot(\theta)$

3.10 Product to Sum Identities

For 2 angles, there exists Product to Sum and Sum to product identities

$sin(u)cos(v) = \frac{sin(u+v)+sin(u-v)}{2}$

$sin(u)sin(v) = \frac{cos(u-v)-cos(u+v)}{2}$

$cos(u)cos(v) = \frac{cos(u-v)+cos(u+v)}{2}$

$cos(u)sin(v) = \frac{sin(a+b)-sin(a-v)}{2}$

$cos(u+v) = cos(u)cos(v) - sin(u)sin(v)$

$cos(u-v) = cos(u)cos(v) + sin(u)sin(v)$

$sin(u+v) = sin(u)cos(v) + cos(u)sin(v)$

$sin(u-v) = sin(u)cos(v) - cos(u)sin(v)$

$tan(a+b) = \frac{tan(a)+tan(b)}{1-tan(a)tan(b)}$

$tan(a-b) = \frac{tan(a)-tan(b)}{1+tan(a)tan(b)}$

$cos(u)cos(v) = \frac{cos(u+v)+cos(u-v)}{2}$

$sin(u)sin(v) = \frac{cos(u-v)-sin(u+v)}{2}$

$sin(u)cos(v) = \frac{sin(u+v)+sin(u-v)}{2}$

$cos(u)sin(v) = \frac{sin(u+v)-cos(u-v)}{2}$

$sin(u) + sin(v) = 2sin(\frac{u+v}{2})cos(\frac{u-v}{2})$

$sin(u) - sin(v) = 2cos(\frac{u+v}{2})sin(\frac{u-v}{2})$

$cos(u) + cos(v) = 2cos(\frac{u+v}{2})cos(\frac{u-v}{2})$

$cos(u) - cos(v) = 2sin(\frac{u+v}{2})sin(\frac{u-v}{2})$

3.11 Double Angle Identities

$cos(2A) = 1 - 2sin^2(A)$

$cos(2A) = cos^2(A) - sin^2(A)$

$cos(2A) = 2cos^2(A) - 1$

$sin(2A) = 2sin(A)cos(A)$

$tan(2A) = \frac{2tan(A)}{1-tan^2(A)}$

3.12 Half Angle Identities

$cos(\frac{u}{2}) = \pm\sqrt{\frac{1-cos(u)}{2}}$

$sin(\frac{u}{2}) = \pm\sqrt{\frac{1-cos(u)}{2}}$

$tan(\frac{u}{2}) = \pm\sqrt{\frac{1-cos(u)}{1+cos(u)}}$

$tan(\frac{u}{2}) = \frac{sin(u)}{1+cos(u)}$

$tan(\frac{u}{2}) = \frac{1-cos(u)}{sin(u)}$

3.13 Side and Angle Relations

Side Angle Side

$A = \frac{1}{2}bcsin(A)$

$A = \frac{1}{2}absin(C)$

$A = \frac{1}{2}acsin(B)$

Angle Side Angle

Side Side Side

$A_1 = \frac{s_2^2+s_3^2-s_1^2}{2s_2s_3}$

$A_2 = \frac{s_1^2+s_3^2-s_2^2}{2s_1s_3}$

$A_3 = \frac{s_1^2+s_2^2-s_3^2}{2s_1s_2}$

Law of Sines

$\frac{a}{sin(a)} = \frac{b}{sin(b)} = \frac{c}{sin(c)}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc * cos(A)$

$b^2 = a^2 + c^2 - 2ac * cos(B)$

$c^2 = a^2 + b^2 - 2ab * cos(C)$

3.14 Herons Formula

Semi-Perimeter $s = \frac{a+b+c}{2}$

Area $\Rightarrow A = \sqrt{s(s-a)(s-b)(s-c)}$

3.15 Arc Length and Sectors of a Circle

Arc Length $\Rightarrow s = r\theta$ where r = radius of the circle and θ is the angle formed by the two lines that connect to the endpoints of the arc

Sector Area $\Rightarrow A = \frac{r^2\theta}{2}$

3.16 Bearing

Bearing is in the form [NSEW]θ°[NSEW] where the first letter is where you are traveling from and the second letter is where you are going to.

Directional Symbol	Direction
N	North
S	South
E	East
W	West

3.17 Cofunction Identities

The cofunction is a complementary function in trigonometry.

3.17.1 Cofunction Identities for Degrees

Function	Cofunction
$sin(\theta)$	$cos(90° - \theta)$
$cos(\theta)$	$sin(90° - \theta)$
$tan(\theta)$	$cot(90° - \theta)$
$csc(\theta)$	$sec(90° - \theta)$
$sec(\theta)$	$csc(90° - \theta)$
$cot(\theta)$	$tan(90° - \theta)$

3.17.2 Cofunction Identities for Radians

Function	Cofunction
$sin(\theta)$	$cos(\frac{\pi}{2} - \theta)$
$cos(\theta)$	$sin(\frac{\pi}{2} - \theta)$
$tan(\theta)$	$cot(\frac{\pi}{2} - \theta)$
$csc(\theta)$	$sec(\frac{\pi}{2} - \theta)$
$sec(\theta)$	$csc(\frac{\pi}{2} - \theta)$
$cot(\theta)$	$tan(\frac{\pi}{2} - \theta)$

4 Pre-Algebra

PRE-ALGEBRA Calculators

4.1 Place Value and Notation

4.1.1 Place Value

The Place Value is as follows for the number 123,456,789

1	2	3	4	5	6	7	8	9
hundred-million	ten-million	million	hundred-thousand	ten-thousand	thousand	hundred	ten	one

For Decimals, the The Place Value is as follows for the decimal 0.12345678

1	2	3	4	5	6	7	8
tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths	ten-millionths	hundred-millionths

4.1.2 Expanded Notation

The number 123,456,789 using expanded notation is as follows:

$1 * 10^8 + 2 * 10^7 + 3 * 10^6 + 4 * 10^5 + 5 * 10^4 + 6 * 10^3 + 7 * 10^2 + 8 * 10^1 + 9 * 10^0$

Each number at decimal place (p) n_p is written as $10 * n_p^{p-1}$

4.1.3 Word Notation

Word notation is the verbal expression of a number using the following translations:

hundred-million	ten-million	million	hundred-thousand	ten-thousand	thousand	hundred	ten	one
1	2	3	4	5	6	7	8	9

4.1.4 Standard Notation

The number a(1000) + b(100) + c(10) + d(1) is written in standard notation

4.2 Proportions

Proportions: $\frac{a}{b} = \frac{c}{d} \Rightarrow a * d = b * c$

4.3 Order of Operations PEDMAS

PEDMAS

P	Parentheses	()
E	Exponent	x^n
D	Division	$\div$
M	Multiplication	$*$
A	Addition	$+$
S	Subtraction	$-$

4.4 Factoring and Divisibility

4.4.1 Divisibility

Number	Divisibility Rules
2	Last Digit of the number is 0
3	sum of it's digits ends is divisible by 3
4	last two digits are divisible by 4
5	The number ends with a 0 or 5
6	If it is divisible by both 2 and 3
7	multiply each respective digit by 1,3,2,6,4,5 working backwards and repeat as necessary
8	Last 3 digits are divisible by 8
9	Sum of the digits are divisible by 9
10	Last Digit of the number is 0
11	Sum of odd digits Sum of Even Digits is either 0 or Divisible by 11

4.4.2 Greatest Common Factor and Least Common Multiple

The Greatest Common Factor is found by taking all of the factors of a group of numbers, finding the common factors, and choosing the highest factor in common of all numbers.

The Least Common Multiple is found by taking all of the multiples of a group of numbers, finding the common multiples, and choosing the highest multiple in common of all numbers.

4.4.3 Prime Factorization and Prime Power Decomposition

The prime factorization of a number is expressing a number in terms of the product of primes

The Prime Power Decomposition is the prime factorization grouped by factor.

4.4.4 Greatest Common Factor Rewrite Sum

1. Using the distributive property, rewrite 12 + 15 by factoring out the GCF
2. Find the minimum of your two numbers $\Rightarrow Min(12, 15) = 12$
3. Factors of 12 $\Rightarrow 1, 2, 3, 4, 6, 12$
4. Factors of 15 (up to our minimum of 12) $\Rightarrow 1, 3, 5$
5. Greatest Common Factor of both lists is 3
6. Divide 12 by 3 $\frac{12}{3} = 4$ to get s_1
7. Divide 15 by 3 $\frac{15}{3} = 5$ to get s_2
8. Rewrite using the formula: $GCF(s_1 + s_2)$
9. $3(4 + 5)$
10. Check our work: $3(4 + 5) = 3(9) = 27 = 12 + 15$

4.5 Fractions

4.5.1 Fraction Definition

A fraction is written as $\frac{a}{b}$ where:

- a is the numerator
- b is the denominator
- the vertical line is the vincula

4.5.2 Classify Fractions

To classify a fraction, we compare the numerator and the denominator:

1. If the numerator is less than the denominator, we have a proper fraction
2. If the numerator is greater than or equal to the denominator, we have an improper fraction

$\frac{3}{5} \Rightarrow$ Proper Fraction

$\frac{9}{7} \Rightarrow$ Improper Fraction

4.5.3 Operations with common denominators

Operation	Description
Addition	$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
Subtraction	$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$
Multiplication	$\frac{a}{b} * \frac{c}{d} = \frac{ac}{bd}$
Division	$\frac{a}{b} \div \frac{c}{b} = \frac{a}{b} * \frac{d}{c} = \frac{ad}{bc}$

4.5.4 Mixed Number to Improper Fraction

For a mixed number $a\frac{b}{c}$, we convert to an improper fraction as follows:

$\frac{c*a+b}{c}$

$1\frac{2}{3} \Rightarrow \frac{3*1+2}{3} \Rightarrow \frac{5}{3}$

4.5.5 Reciprocal

The Reciprocal of a fraction $\frac{a}{b} = \frac{b}{a}$

4.5.6 Unit Fraction

For a fraction $\frac{a}{b}$, the unit fraction is denoted as $\frac{1}{b}$

4.5.7 Equivalent Fraction

The Equivalent Fraction of $\frac{a}{b} \Longrightarrow$ Multiply top and bottom by any integer n $\frac{an}{bn}$

4.6 Percentage and Decimals

Expressing $\frac{a}{b}$ as a Decimal is equivalent to multiplying by 100 percent $\Rightarrow \frac{100*a}{b}$

Convert a decimal d with (n) decimal places to a fraction is done by multiplying by $\frac{n*10^d}{10^d}$

Convert the percent p to a fraction and decimal $\frac{p}{100}$

4.7 Interval Counting

$a(b)c$ is read as count a to c in intervals of b

20(10)100 reads count from 20 to 100 in intervals of 10

$20, 30, 40, 50, 60, 70, 80, 90, 100$

4.8 Ratios

Ratios

$a : b = \frac{a}{b}$

$a : b$ for c $= \frac{ac}{a+b}$

$a : b$ and $c : d \Rightarrow \frac{ac}{bd}$

4.9 Digit Sum

1. Take a number abc.
2. The digit sum is $a + b + c$
3. If the digit sum is greater than 9, add repeat step 2
4. The reduced digit sum is when the digit sum is less than 10

Take 987654. Digit Sum is $9 + 8 + 7 + 6 + 5 + 4 = 39$

$39 \Rightarrow 3 + 9 = 12$

$12 \Rightarrow 1 + 2 = 3$

4.10 Exponents

Exponent Rules

$x^0 = 1$

$x^2 = x \cdot x$

$x^3 = x \cdot x \cdot x$

$x^n = x \cdot x \cdot x \cdots$ n times

$x^{-n} = \frac{1}{x^n}$

$b^m \cdot b^n = b^{m+n}$

$b^{\frac{m}{n}} = b^{m-n}$

$(b^m)^n = b^{mn}$

$(bc)^n = b^n c^n$

$(\frac{b}{c})^n = \frac{b^n}{c^n}$

4.10.1 Simplest Exponent Form

Take $3 * a * a * a * b * b$. We want to group all variables that occur more than once using exponents.

We have three a's and two b's. So we take our constant of 3 and then group our variables by exponents.

$3a^3b^2$

4.11 Square Roots and Roots

$\sqrt{x^2} = x$

$\sqrt[n]{x} = x^{\frac{1}{n}}$

- $\sqrt{1} = 1$
- $\sqrt{4} = 2$
- $\sqrt{9} = 3$
- $\sqrt{16} = 4$
- $\sqrt{25} = 5$
- $\sqrt{36} = 6$
- $\sqrt{49} = 7$
- $\sqrt{64} = 8$
- $\sqrt{81} = 9$
- $\sqrt{100} = 10$

4.11.1 Bakshali Method for Square Roots

1. Find the square root of a number S
2. Starting at 1, square each positive integer until it's total exceeds S. Call the final positive integer N
3. Calculate d $\Rightarrow d = S - N^2$
4. $P = \frac{d}{2N}$
5. $A = N + P$
6. $\sqrt{S} \approx A - \frac{P^2}{2A}$

4.11.2 Newtons Method for Square Roots

1. The square root of a number can be represented by the function $f(x) = x^2 - S$
2. Taking the Derivative of this, we have $f'(x) = 2x$
3. Since the square root is always positive, we start with $x_0 = 1$
4. $x_1 = x_0 + ((x_0) - S)/'(x_0)$
5. Repeat by using $x_n = x_{n-1} + ((x_{n-1}) - S)/'(x_{n-1})$
6. You can stop when the difference between the current and last iteration are close

4.11.3 Babylonian Method for Square Roots

Start with i = 0, and iterate until the desired result is reached:

$x_i = \frac{1}{2}(x_{i-1} + \frac{S}{x_{i-1}})$

4.11.4 Exponential Identity Method for Square Roots

1. To find the square root of a number S $\Rightarrow \sqrt{S}$
2. Evaluate $e^{\frac{Ln(S)}{2}}$

4.12 Algebraic Expressions

Phrase	Sign to use	Example	Algebraic Expressions
Plus	+	6 plus 3	6 + 3
Added to	+	6 added to 3	6 + 3
Increased by	+	6 increased by 3	6 + 3
And	+	6 and 3	6 + 3
Minus	-	6 minus 3	6 - 3
Subtracted by	-	6 subtracted by 3	6 - 3
Decreased by	-	6 decreased by 3	6 - 3
Reduced by	-	6 reduced by 3	6 - 3
Less	-	6 less 3	6 - 3
Diminished by	-	6 diminished by 3	6 - 3
Times	*	6 times 3	6 * 3
Multiplied by	*	6 multiplied by 3	6 * 3
The product of	*	The product of 6 and 3	6 * 3
Divided by	/	6 divided by 3	6 / 3
Over	/	6 over 3	6 / 3
The quotient of	/	The quotient of 6 over 3	6 / 3
Equals	=	x equals 6	x = 6
Is equal to	=	x is equal to 6	x = 6
Is the same as	=	x is the same as 6	x = 6
Is	=	x is 6	x = 6
Does not equal	≠	x does not equal 6	x ≠ 6
Is not equal to	≠	x is not equal to 6	x ≠ 6
Is not the same as	≠	x is not the same as 6	x ≠ 6
Is not	≠	x is not 6	x ≠ 6

4.12.1 Coefficient Phrases

Phrase	Multiplier to use	Example	Algebraic Expressions
Twice	2	Twice x	2x
Double	2	Double x	2x
Thrice	3	Thrice x	3x
Triple	3	Triple x	3x
Quadruple	4	Quadruple x	4x
Half	$\frac{1}{2}$	Half x	$\frac{x}{2}$
Negative	-1	Negative x	-x

4.12.2 Exponents

- Squared means a power of 2 $\Rightarrow$ x squared $\Rightarrow$ x^2
- Cubed means a power of 3 $\Rightarrow$ x cubed $\Rightarrow$ x^3
- Raise x to the y power means an exponent: x^y

5 Algebra

ALGEBRA Calculators

5.1 1 unknown equations

5.1.1 One-Step Equations

One step equations such as $cx = d \Rightarrow 2x = 90$

All you do is take the right side of the equation and divide it by the coefficient of the variable. So $x = \frac{d}{c}$

5.1.2 Two-Step Equations

Two-step equations such as $ax - b = c \Rightarrow 2x - 9 = 31$

1. Add or subtract to remove the constant from the left hand side
2. Then divide the right hand side of the equation by the coefficient of the variable

For $ax - b = c \Rightarrow x = \frac{c+b}{a}$

For $ax + b = c \Rightarrow x = \frac{c-b}{a}$

5.2 Binomials

FOIL - First Outside Inside Last

$(a + b)(c + d) = (a * c) + (b * c) + (a * d) + (b * d)$

Difference of two squares

$a^2x^2 - b^2y^2 = (ax + by)(ax - by)$

5.3 Imaginary Numbers

The imaginary Number i $\Rightarrow i = \sqrt{-1}$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = (i\sqrt{-1})^2 = -i$

$i^4 = ((\sqrt{-1})^2)^2 = (-1)^2 = 1$

The goal here is to break down the powers. First step is getting power multiples of 4 since $i^4 = 1$

5.4 Complex Number Operations

Given 2 complex numbers $a + bi$ and $c + di$

Adding: $a + bi + (c + di) = (a + c) + (b + d)i$

Subtracting: $a + bi - (c + di) = (a - c) + (b - d)i$

Multiplying: $(a + bi)(c + di) = ac + adi + bdi + bdi^2$

Dividing: $\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$

Square Root: $\sqrt{a + bi}$ has two roots:

$r = \sqrt{a^2 + b^2}$

$y = \sqrt{\frac{1}{2}(r - a)}$

$x = \frac{b}{2y}$

Root 1: $x + yi$

Root 2: $-x - yi$

Absolute Value: $|a + bi| = \sqrt{a^2 + b2}$

5.5 Intersection of 2 lines

2 lines are either parallel, perpendicular, or not intersecting.

Parallel $\Rightarrow m_1 = m_2$ where m_1 is the slope of line 1 and m_2 is the slope of line 2

Perpendicular $\Rightarrow m_1 = \frac{-1}{m_2}$

If the two lines are not parallel or perpendicular, the lines intersect

The angle between the 2 lines denoted as $\theta \Rightarrow tan(\theta) = \frac{m_2 - m_1}{1 + m_2 m_1}$

5.6 Quadratic Equations

The solution to the Quadratic Equation $ax^2 + bx + c = 0$ is:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If the discriminant $b^2 - 4ac > 0$, 2 real unequal roots exist

If the discriminant $b^2 - 4ac = 0$, 1 real root exists

If the discriminant $b^2 - 4ac < 0$, complex conjugate roots exist

Vertex of the parabola formed by the quadratic is $(\frac{-b}{2a}, f(\frac{-b}{2a}))$ where $y = a(x - h)^2 + k$

Concavity is the x^2 coefficient, Up if positive, down if negative.

5.7 3 Point Quadratic Equation

Take 3 points: $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

- $a = x_1^2$
- $b = x_1$
- $c = 1$
- $d = y_1$
- $e = x_2^2$
- $f = x_2$
- $g = 1$
- $h = y_2$
- $i = x_3^2$
- $j = x_3$
- $k = 1$
- $l = y_3$

$\Delta = (a * f * k) + (b * g * i) + (c * e * j) - (c * f * i) - (a * g * j) - (b * e * k)$

$a = \frac{(d*f*k)+(b*g*l)+(c*h*j)-(c*f*l)-(d*g*j)-(b*h*k)}{\Delta}$

$b = \frac{a*h*k)+(d*g*i)+(c*e*l)-(c*h*i)-(a*g*l)-(d*e*k)}{\Delta}$

$c = \frac{(a*f*l)+(b*h*i)+(d*e*j)-(d*f*i)-(a*h*j)-(b*e*l)}{\Delta}$

5.8 Descartes Rule of Signs

$f(x) = 2x3 - 7x2 + 4x - 14$

1. Sign Change + to -
2. Sign Change - to +
3. Sign Change + to -

3 roots - 1 pair (2 roots) = 1

Therefore, we have a possible combination of (3 or 1) positive roots

5.9 Slope and Line Equations

Slope and Line Equations with 2 points

Given two standard points on a Cartesian Graph of $(x_1, y_1), (x_2, y_2)$

Standard Equation of a line is $y = mx + b$ where $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-Slope Format is $y - y_1 = m(x - x_1)$

Distance between the points is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Rational Exponents - Fractional Indices

$x^{\frac{a}{b}} = (\sqrt[b]{x})^a$

5.10 System of Equations 2 unknowns

Substitution or Elimination

5.11 Cramers Rule

5.11.1 2 unknowns

Cramers Rule for 2 unknowns Determinant Δ

Equation 1: $ax + by = c$

Equation 2: $dx + ey = f$

Build the matrix of coefficients

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

$\Delta = a * e + b * d$

$x = \frac{c*e - b*f}{\Delta}$

$y = \frac{a*f - c*d}{\Delta}$

Augmented matrix AX = B

$$A = \begin{vmatrix} a & b \\ d & e \end{vmatrix} X = \begin{vmatrix} x \\ y \end{vmatrix} B = \begin{vmatrix} c \\ f \end{vmatrix}$$

$$A|B = \begin{vmatrix} a & b & | & c \\ d & e & | & f \end{vmatrix}$$

5.11.2 3 unknowns

Cramers Rule for the following system of equations:

$ax + by + cz = d$

$ex + fy + gz = h$

$ix + jy + kz = l$

$\Delta = (a * f * k) + (b * g * i) + (c * e * j) - (c * f * i) - (a * g * j) - (b * e * k)$

$x1 = (d * f * k) + (b * g * l) + (c * h * j) - (c * f * l) - (d * g * j) - (b * h * k)$

$y1 = (a * h * k) + (d * g * i) + (c * e * l) - (c * h * i) - (a * g * l) - (d * e * k)$

$z1 = (a * f * l) + (b * h * i) + (d * e * j) - (d * f * i) - (a * h * j) - (b * e * l)$

$x = \frac{x1}{\Delta}$

$y = \frac{y1}{\Delta}$

$z = \frac{z1}{\Delta}$

Augmented matrix AX = B

$$A = \begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix} X = \begin{vmatrix} x \\ y \\ z \end{vmatrix} B = \begin{vmatrix} d \\ h \\ l \end{vmatrix}$$

$$A|B = \begin{vmatrix} a & b & c & | & d \\ d & e & f & | & g \\ h & i & j & | & k \end{vmatrix}$$

5.11.3 4 unknowns

Cramers Rule for the following system of equations:

$aw + bx + cy + dz = e$

$fw + gx + hy + iz = j$

$kw + lx + my + nz = o$

$pw + qx + ry + sz = t$

$\Delta_1 = (agms) + (ahnq) + (ailr) - (aimq) - (agnr)$

$\Delta_2 = -(ahls) - (fbms) - (fcnq) - (fdlr) + (fdmq)$

$\Delta_3 = (fbnr) + (fcls) + (kbhs) + (kciq) + (kdgr)$

$\Delta_4 = -(kdhq) - (kbir) - (kcgs) - (pbhn) - (pcil)$

$\Delta_5 = -(pdgm) + (pdhl) + (pbim) + (pcgn)$

$\Delta = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5$

$W1 = (egms) + (ehnq) + (eilr) - (eimq) - (egnr)$

$W2 = -(ehls) - (jbms) - (jcnq) - (jdlr) + (jdmq)$

$W3 = (jbnr) + (jcls) + (obhs) + (ociq) + (odgr)$

$W4 = -(odhq) - (obir) - (ocgs) - (tbhn) - (tcil)$

$W5 = -(tdgm) + (tdhl) + (tbim) + (tcgn)$

$W = W1 + W2 + W3 + W4 + W5$

$X1 = (ajms) + (ahnt) + (aior) - (aimt) - (ajnr)$

$X2 = -(ahos) - (fems) - (fcnt) - (fdor) + (fdmt)$

$X3 = (fenr) + (fcos) + (kehs) + (kcit) + (kdjr)$

$X4 = -(kdht) - (keir) - (kcjs) - (pehn) - (pcio)$

$X5 = -(pdjm) + (pdho) + (peim) + (pcjn)$

$X = X1 + X2 + X3 + X4 + X5$

$Y1 = (agos) + (ajnq) + (ailt) - (aioq) - (agnt)$

$Y2 = -(ajls) - (fbos) - (fenq) - (fdlt) + (fdoq)$

$Y3 = (fbnt) + (fels) + (kbjs) + (keiq) + (kdgt)$

$Y4 = -(kdjq) - (kbit) - (kegs) - (pbjn) - (peil)$

$Y5 = -(pdgo) + (pdjl) + (pbio) + (pegn)$

$Y = Y1 + Y2 + Y3 + Y4 + Y5$

$Z1 = (agmt) + (ahoq) + (ajlr) - (ajmq) - (agor)$

$Z2 = -(ahlt) - (fbmt) - (fcoq) - (felr) + (femq)$

$Z3 = (fbor) + (fclt) + (kbht) + (kcjq) + (kegr)$

$Z4 = -(kehq) - (kbjr) - (kcgt) - (pbho) - (pcjl)$

$Z5 = -(pegm) + (pehl) + (pbjm) + (pcgo)$

$Z = Z1 + Z2 + Z3 + Z4 + Z5$

$w = \frac{W}{\Delta}$

$x = \frac{X}{\Delta}$

$y = \frac{Y}{\Delta}$

$z = \frac{Z}{\Delta}$

Augmented matrix AX = B

$$\text{A} = \begin{vmatrix} \text{a} & \text{b} & \text{c} & \text{d} \\ \text{f} & \text{g} & \text{h} & \text{i} \\ \text{k} & \text{l} & \text{m} & \text{n} \\ \text{p} & \text{q} & \text{r} & \text{s} \end{vmatrix} \text{X} = \begin{vmatrix} \text{w} \\ \text{x} \\ \text{y} \\ \text{z} \end{vmatrix} \text{B} = \begin{vmatrix} \text{e} \\ \text{j} \\ \text{o} \\ \text{t} \end{vmatrix}$$

$$\text{A|B} = \begin{vmatrix} \text{a} & \text{b} & \text{c} & \text{d} & | & \text{e} \\ \text{f} & \text{g} & \text{h} & \text{i} & | & \text{j} \\ \text{k} & \text{l} & \text{m} & \text{n} & | & \text{o} \\ \text{p} & \text{q} & \text{r} & \text{s} & | & \text{t} \end{vmatrix}$$

5.12 Cubic Equations

$ax^3 - bx^2 - cx + d = 0$

$\Delta = 4b^3d - b^2c^2 + 4ac^3 - 18abcd + 27a^2d^2$

$f = \frac{(3c/a) - b^2/a^2}{3}$

$g = \frac{2b3/a3 - 9bc/a2 + 27d/a}{27}$

$h = \frac{g^2}{4} + \frac{f^3}{27}$

$i = \sqrt{0.25g^2 - h}$

$j = i^{1/3}$

$k = Arccosine(\frac{-g}{2i})$

$l = -j$

$m = Cos(\frac{k}{3})$

$n = \sqrt{3} * Sin(\frac{k}{3})$

$p = \frac{-b}{3a}$

Real Roots are (x_1, x_2, x_3)

$x_1 = 2j * Cos(\frac{k}{3}) - \frac{b}{3a}$

$x_2 = l(m + n) + p$

$x_3 = l(m - n) + p$

5.13 Quartic Equations

$ax^4 + bx^3 - cx^2 - dx + e = 0$

$f = c - \frac{3b^2}{8}$

$g = d + \frac{3b^3}{8} - \frac{bc}{2}$

$h = e - \frac{3b^4}{256} + \frac{b^2c}{16} - \frac{bd}{4}$

Using our f, g, and h values, we form the following cubic equation:

$x^3 + \frac{fx^2}{2} + \frac{f^2-4h}{16}x - \frac{g^2}{64} = 0$

Then, use the cubic formula above to break down the cubic into roots

5.14 Scientific Notation

The Scientific Notation for a number is denoted as $a * (10^n)$ where $1 \leq a < 10$

5.15 Intercepts

The X and Y intercepts for $ax + by = c$

Y Intercept, isolate Y, and set x to 0

$by = c - ax \Rightarrow \frac{by}{b} = \frac{c-a(0)}{b} \Rightarrow y = \frac{c}{b}$

X Intercept, isolate X, and set y to 0

$ax = c - by \Rightarrow \frac{ax}{a} = \frac{c-b(0)}{a} \Rightarrow x = \frac{c}{a}$

5.16 Expanding Polynomials

5.16.1 Binomial Theorem

Expanding $(a + b)^n = a^n +_n P_1 a^{n-1}b +_n P_2 a^{n-2}b^2 + \cdots +_n P_k a^{n-k}b^k + \cdots + b^n$

5.17 Factoring Polynomials

Difference of Cubes Factoring $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Sum of Cubes Factoring $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

5.18 Synthetic Division

Synthetic Division is used to determined roots.

5.19 Variation Equations

5.19.1 Direct Variation Equations

Variation Equation Type	Relation Equation
Direct Variation	$y = kx$
Inverse Variation	$y = \frac{k}{x}$
Squared Variation	$y = kx^2$
Cubed Variation	$y = kx^3$
Square Root Variation	$y = k\sqrt{x}$
Inverse of the Square Variation	$y = \frac{k}{x^2}$
Inverse of the Cube Variation	$y = \frac{k}{x^3}$
Inverse of the Square Root Variation	$y = \frac{k}{\sqrt{x}}$

5.19.2 Joint Variation Equations

Variation Equations Example:

a varies jointly with b and c, and $a = 12$ when $b = 1$ and $c = 6$, solve for a when $b = 2, c = 3$ with a relationship of $a = kbc$

6 Pre-Calculus

PRE-CALCULUS

The Antilog of a using base b is b^a

Base Conversions

Conjugates

To simplify an expression such as $\frac{1}{a+\sqrt{b}}$ we multiply the numerator and denominator by the conjugate $a-\sqrt{b}$

$\frac{a-\sqrt{b}}{(a+\sqrt{b})(a-\sqrt{b})} = \frac{a-\sqrt{b}}{a^2-b^2}$

6.1 Demoivre's Theorem

Demoivres Theorem

Basic

If $z = rcis(\theta) \Rightarrow z^n = r^n cis(n\theta)$

Polar

$z = r(cos(\theta) + isin(\theta))$ where

$a = rcos(\theta)$

$b = rsin(\theta)$

$r = \sqrt{a^2 + b^2}$

6.2 Logarithms

Logarithms

6.2.1 Properties

If $b^y = x \Rightarrow log_b(x) = y$

$log_b xy = log_b x + log_b y$

$log_b(\frac{x}{y}) = log_b x - log_b y$

$log_b(x) = \frac{log_x}{log_b}$

Change of Base

$log_b(x) = \frac{Ln(x)}{Ln(b)}$

6.2.2 Natural Log

$Ln(e^x) = e^{Ln(x)} = x$

$Ln(x^n) = n * Ln(x)$

$Ln(xy) = Ln(x) + Ln(y)$

$Ln(\frac{x}{y}) = Ln(x)Ln(y)$

6.3 Conics

6.3.1 Circles

Standard Form Equation of a circle is $(x-h)^2+(y-k)^2=r^2$

General Form Equation of a circle is $x^2+y^2-2hx-2ky+h^2+k^2-r^2=0$

where (h,k) is the center of the circle and r is the radius.

Chord Length $=2\sqrt{r2-t2}$ where r = radius and t = Circle Center to Chord Midpoint Distance

6.3.2 Hyperbolas

Standard Equation of a Horizontal Hyperbola is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Asymptotes are $y=\pm\frac{b}{a}x$

Standard Equation of a Vertical Hyperbola is $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$

Asymptotes are $y=\pm\frac{a}{b}x$

Foci at $(0,c)$ and $(0,-c)$ where $c=\sqrt{a^2+b^2}$

Eccentricity: $\varepsilon=\frac{\sqrt{a^2-b^2}}{a}$

Latus Rectum: $LR=\frac{2b^2}{a}$

6.3.3 Parabolas

General equation of the Parabola is $Ax^2+Cy^2+Dx+Ey+F=0$

Vertex at $h,k)$

Standard Vertical Parabola Equation is $(x-h)^2=4c(y-k)$

$$\begin{cases} c>0, & \text{opens up if } c>0 \\ c<0, & \text{opens down if } c<0 \end{cases} \tag{2}$$

Focus at $(0,c)$ and directrix at $y=c$

Standard Horizontal Parabola Equation is $(y-c)^2=4c(x-h)$ then we have

$$\begin{cases} c>0, & \text{opens right if } c>0 \\ c<0, & \text{opens left if } c<0 \end{cases} \tag{3}$$

Focus at $(c,0)$ and directrix at $x=c$

6.3.4 Ellipse

Standard Equation of a Horizontal Ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Standard Equation of a Vertical Ellipse is $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

6.3.5 Polar Conics

The Vertical Directrix Polar Conic Equations where e = eccentricity, d = directrix are denoted as:

$r=\frac{ed}{1-e(cos(\theta))}$

$r=\frac{ed}{1+e(cos(\theta))}$

The Horizontal Directrix Polar Conic Equations where e = eccentricity, d = directrix are denoted as:

$r = \frac{ed}{1-e(sin(\theta))}$

$r = \frac{ed}{1+e(sin(\theta))}$

6.4 Counting Formulas

Factorials $n! = n * (n-1) * (n-2) * \cdots * 1$

Algorithm 6.4.1 Factorial Algorithm

```
procedure FACTORIAL(n)
    answer := n
    while n > 0 do                          ▷ Keep multiplying until we hit 1
        n := n − 1
        answer := answer * n
    end while
    return answer                           ▷ The factorial is the positive integer answer
end procedure
```

Stirling Approximation

$n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$

6.4.1 Permutations

The Permutations formula for choosing k ways from n possibilities is

$_nP_k = \frac{n!}{(n-k)!}$

With permutations, order matters.

6.4.2 Combinations

The Combinations formula for choosing k **unique** ways from n possibilities is

$_nC_k = \frac{n!}{k!(n-k)!}$

With combinations, order does not matter.

Symmetry property: $_nC_k =_n C_{nk}$

Pascals identity: $_{n+1}C_k =_n C_{k1} +_n C_k$

6.4.3 Group Combinations

Group Combinations are found from multiplying each individual combination. A Group/Team consist of 5 men and 7 women. How many Group/Teams of 5 can be formed using 3 men and 2 women?

$C(5,3) * C(7,2)$

6.5 Functions

Function Test

For all the (x, y) pairs, a function exists if there are not more than one x value

Function Test

$f(x) = ax^n + bx^{n-1} + \cdots + c$

7 Calculus

CALCULUS

7.1 Quadrants

Quadrant	(x, y)
I	(x, y)
II	$(-x, y)$
III	$(-x, -y)$
IV	$(x, -y)$

7.2 Point Reflection and Rotation

Reflection Type	Reflected Point
y-axis	$(x, y) \Rightarrow (-x, y)$
x-axis	$(x, y) \Rightarrow (x, -y)$
origin	$(x, y) \Rightarrow (-x, -y)$

Point Rotation

Rotation Degrees	New Point
$90°$	$(x, y) \Rightarrow (-y, x)$
$180°$	$(x, y) \Rightarrow (-x, -y)$
$270°$	$(x, y) \Rightarrow (y, -x)$

7.3 Sequences and Series

7.3.1 Summation Properties

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} 1 = n$$

7.3.2 Series Formulas

Series Type	Explicit Formula
Arithmetic	$a_n = a_1 + (n-1)d$
Geometric	$a_n = a_1(r - n)$
Infinite Geometric	$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$
Harmonic	$a_n = \frac{1}{n}$

7.4 Limits

Constants: $\lim_{x\to a} c = c$

Single Variable: $\lim_{x\to a} x = a$

Coefficients: $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$

Single Variable with Exponent: $\lim_{x\to a} x^n = a^n$

Addition (Limit of a sum is the sum of the limits): $\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$

Subtraction (Limit of a difference is the difference of the limits): $\lim_{x\to a}[f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$

Multiplication (Limit of a product is the product of the limits): $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

Division (Limit of a quotient is the quotient of the limits): $\lim_{x\to a}[\frac{f(x)}{g(x)}] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

Roots: $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$

Two Sided Limit: $\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

Squeeze Theorem or Sandwich Theorem: If $f(x) \leq g(x)$ when x is near a and the limits of $f(x)$ and $g(x)$ both exist as x approaches a, then:

$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$ then $\lim_{x\to a} g(x) = L$

7.4.1 Continuity

A function f is continuous at a number a if the following three conditions hold:

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

A function is continuous from the right at a number a if $\lim_{x\to a^+} f(a)$

A function is continuous from the left at a number a if $\lim_{x\to a^-} f(a)$

The following types of functions are continuous at every number in their domains:

- polynomials
- rational functions
- root functions
- trigonometric functions
- inverse trigonometric functions
- exponential functions
- logarithmic functions

$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x))$

if g is continuous at a and f is continous at g(a), the the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continous at a

7.4.2 Intermediate Value Theorem

if f is continuous on the closed interval [a,b], then let N be any number between $f(a)$ and $f(b)$. A number c exists in (a, b) such that $f(c) = N$

7.4.3 Vertical Asymptote Limit

The line $x = a$ is a vertical asymptote of the curve $y = f(x)$ if one or more of the following statements is true:

- $\lim_{x \to a} f(x) = \infty$
- $\lim_{x \to a^-} f(x) = \infty$
- $\lim_{x \to a^+} f(x) = \infty$
- $\lim_{x \to a} f(x) = -\infty$
- $\lim_{x \to a^-} f(x) = -\infty$
- $\lim_{x \to a^+} f(x) = -\infty$

7.4.4 Horizontal Asymptote Limit

The line $y = L$ is a horizontal asymptote of the curve $y = f(x)$ if either:

- $\lim_{x \to \infty} f(x) = L$
- $\lim_{x \to -\infty} f(x) = L$

7.4.5 Other Limit Laws

$\lim_{x \to \infty} \frac{1}{x^n} = 0$

$\lim_{x \to -\infty} \frac{1}{x^n} = 0$

$\lim_{x \to \infty} e^x = 0$

7.4.6 Eulers Constant Limit

- $e \approx 2.7182818$
- $\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$
- $e = \lim_{n \to \infty}(1 + \frac{1}{n})^n$

7.5 Rates of Change

7.6 Tangent to the curve

$y = f(x)$ at the point $P(a, f(a))$ is a line through P with slope:

$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

With $h = x - a$ and $x = a + h$, we have:

$m = \frac{f(a+h) - f(a)}{h}$

7.7 Instantaneous Rate of Change

$y = f(x)$ and x changes from x_1 to x_2

Then the change, or increment in x is: $\Delta x = x_2 - x_1$ and corresponding change in y is $\Delta y = f(x_2) - f(x_1)$

Average rate of change of y with respect to x: $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

Instantaneous rate of change: $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

7.8 Derivatives

Definition

$f'(x) = \lim_{\delta x \to 0} \frac{(f(x+\Delta x) - f(x))}{\Delta x}$

Constant Rule: $f(x) = c \Rightarrow f'(x) = 0$

Power Rule with no coefficient: $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

Power Rule with coefficient: $f(x) = ax^n \Rightarrow f'(x) = nax^{n-1}$

Exponential General: $f(x) = e^x \Rightarrow f'(x) = e^x$

Exponent Special: $f(u) = e^u \Rightarrow f'(u) = e^u du$

Chain Rule 1: (f(x) and g(x) both differentiable: $F(x) = f \cdot g(x) \Rightarrow F'(x) = f'(g(x))g'(x)$

Chain Rule 2: $y = f(u)$ and $u = g(x) \rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Product Rule: $y = f(x)g(x) \Rightarrow y' = f(x)g'(x) + f'(x)g(x)$

Quotient Rule: $y = \frac{f(x)}{g(x)} \Rightarrow y' = \frac{(f(x)g(x) - g(x)f(x))}{g^2}$

Power Rule and Chain Rule (n is a real number and u is differentiable: $f(x) = u^n \Rightarrow f'(x) = nu^{n-1}u'$

7.8.1 Derivative Rules

- A function is differentiable at a if f'(a) exists
- If f is differentiable at a, then f is continuous at a
- If $f'(x) > 0$ on an interval, then f is increasing on that interval
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval
- If $f''(x) > 0$ on an interval, then f is concave upward on that interval
- If $f''(x) < 0$ on an interval, then f is concave downward on that interval

7.8.2 Trigonometric function derivatives

- $f(x) = sin(x) \Rightarrow f'(x) = cos(x)$
- $f(x) = cos(x) \Rightarrow f'(x) = -sin(x)$
- $f(x) = tan(x) \Rightarrow f'(x) = sec^2(x)$
- $f(x) = csc(x) \Rightarrow f'(x) = -csc(x) \cdot cot(x)$
- $f(x) = sec(x) \Rightarrow f'(x) = sec(x) \cdot tan(x)$
- $f(x) = cot(x) \Rightarrow f'(x) = -csc^2(x)$

7.8.3 Parametric Curve Tangents

Parametric equation curves: $x = f(t), y = g(t)$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

7.8.4 Inverse Trigonometric function derivatives

- $f(x) = sin^{-1}(x) \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
- $f(x) = cos^{-1}(x) \Rightarrow f'(x) = \frac{-1}{\sqrt{1-x^2}}$
- $f(x) = tan^{-1}(x) \Rightarrow f'(x) = \frac{1}{1+x^2}$

- $f(x) = csc(x) \Rightarrow f'(x) = \frac{-1}{|x|\sqrt{x^2-1}}$
- $f(x) = sec(x) \Rightarrow f'(x) = \frac{1}{|x|\sqrt{x^2-1}}$
- $f(x) = cot(x) \Rightarrow f'(x) = \frac{-1}{1+x^2}$

7.8.5 Logarithmic function derivatives

- $f(x) = log_a x \Rightarrow f'(x) = \frac{1}{x \cdot Ln(a)}$
- $f(x) = Ln(x) \Rightarrow f'(x) = \frac{1}{x}$
- $f(x) = Ln(u) \Rightarrow f'(x) = \frac{1}{u}\frac{du}{dx}$
- $f(x) = Ln(|x|) \Rightarrow f'(x) = \frac{1}{x}$

7.9 Hyperbolic Functions

Hyperbolic sine: $sinh(x) = \frac{e^x - e^{-x}}{2}$

Hyperbolic cosine: $cosh(x) = \frac{e^x + e^{-x}}{2}$

Hyperbolic tangent: $tan(x) = \frac{sinh(x)}{cosh(x)} = \frac{1-e^{-2x}}{1+e^{-2x}}$

Hyperbolic secant: $sec(x) = \frac{1}{cosh(x)} = \frac{2}{e^x + e^{-x}}$

Hyperbolic cosecant: $csc(x) = \frac{1}{sinh(x)} = \frac{2}{e^x - e^{-x}}$

Hyperbolic cotangent: $coth(x) = \frac{1}{tanh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

7.10 Taylor Polynomial

n-th degree polynomial of f centered at a: $T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f'^n(a)}{2}(x-a)^n$

7.11 Maximum and Minimum

- A function f has a local maximum or local minimum at c if $f(c) \geq f(x)$ when x is near c
- Fermat's Theorem: If f has a local maximum at c and if $f'(c)$ exists, then $f'(c) = 0$
- If f has a local maximum or minimum at c, then c is a critical number of f

7.12 L'Hospitals Rule

If f and g are differentiable and $g'(x) \neq 0$ near a:

- $\lim_{x\to a} f(x) = 0$
- $\lim_{x\to a} g(x) = 0$
- $\lim_{x\to a} f(x) = \pm\infty$
- $\lim_{x\to a} g(x) = \pm\infty$

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

Summary: Limit of a quotient of functions equals limit of the quotient of derivatives

7.13 Integrals

Definition $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\Delta x$

Power Rule $\int ax^n dx = \frac{ax^{n+1}}{n+1}$

8 Statistics

8.1 Probability

8.1.1 Probability Axioms

- We start with a sample space denoted Ω or S, which is the set of all possible outcomes
- P(Ω) = 1
- We have an event, which is a subset of a sample space
- Probability is assigned to an event, which is the chance that occurs in the entire sample space
- $P(\emptyset) = 0$
- We have nonnegativity of probabilities, so for an event A, we have $P(A) \geq 0$
- Since the sample space is the set of all possible outcomes, we have $P(\Omega) = 1$
- Complement: The probability of anything but A is $P(A^c) = 1 - P(A) \Rightarrow P(A) = 1 - P(A^c)$
- Combine the sample space, one event and an event complement, we have $P(\Omega) = P(A \cup A^c) = 1$
- Additivity: if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$
- $if A \subset B, B = A \cup (A^c \cap B)$
- $A \subset B \Rightarrow P(A) \leq P(B)$
- $A = A \cap (B \cup B^c) = (A \cap B) \cup (A \cap B^c)$

8.1.2 Bayes Rule and Conditional Probability

Bayes Rule $P(B|A) = \frac{P(A \cap B)}{P(A \cap B) + P(A \cap B^c)}$

$A_1, A_2, ..., A_n$ are sets in S where $A_i \cap A_j = \emptyset$ for $i \neq j$

$P(B) = P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_n)$

$P(B|A) = \frac{P(A|B)*P(B)}{P(A|B)*P(B) + P(A|B^c)*P(B^c)}$

$P(A_i \cap B) = P(B|A_i) \cdot P(A_i)$

If A and B are mutually exclusive, they cannot occur at the same time. Also, they are not independent, so:

Event	Independent Events Probability	Mutually Exclusive Events Probability
$P(A \cap B)$	$P(A) \cdot P(B)$	0
$P(B\|A)$	$P(B)$	0
$P(A\|B)$	$P(A)$	0

8.1.3 Bayes Theorem Table Method (3 Events)

A_i	$P(A_i)$	$P(B\|A_i)$	$P(A_i \cap B)$	$P(A_i\|B)$
A_1				
A_2				
A_3				
Total	1.0			

8.1.4 Fundamental Rule of Counting

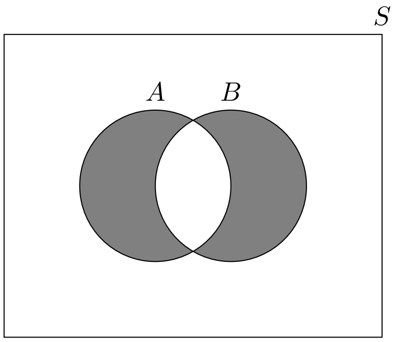
Fundamental Rule of Counting states that the total number of ways one can do something is with k items is $n_1 \cdot n_2 \cdot ... \cdot n^k$

8.1.5 De Morgan Laws

- $(A \cap B)^C = A^C \cup B^C$
- $(A \cup B)^C = A^C \cap B^C$

8.2 Venn Diagram

The Venn Diagram is a way to show the intersection of 2 or more events. The white area below represents $A \cap B$



8.2.1 Law of Total Probability

$P(B) = P(B \cap A_1) + P(B \cap A_2) + \cdots + P(B \cap A_n) = \sum_{i=1}^{n} P(B \cap A_i)$

Two Event Example: $P(B) = P(A \cap B) + P(A^C \cap B)$

$P(B) = P(A) \cdot P(B|A) + P(A^C) \cdot P(B|A^C)$

8.2.2 Event Probabilities

Postulate I: $P(A) \geq 0$

Postulate II: Sample Space S, $P(S) = 1$

Postulate III (Mutually Exclusive Events): $P(A_1 \cup A_2 \cup PA_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$

Empty Set: $P(\emptyset) = 0$ for Sample Space S

$P(E) = \frac{FavorableOutcomes}{TotalOutcomes}$

8.3 Tabular Display

Tabular Display to test if a valid sample space. $\sum_{x=1}^{n} p_x = 1$

x	x_1	x_2	x_3	$\cdots$	x_n
p(x)	$p(x_1)$	$p(x_2)$	$p(x_3)$	$\cdots$	$p(x_n)$

Expected value: $E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \cdots + x_n p(x_n)$

8.4 2 Events

2 Number Sets: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$(A \cap B) \subset (A \cup B)$

If A and B are mutually exclusive (never happen together), then: $PA \cap B) = 0$

8.5 3 Events

3 Events: $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8.6 Odds Probability

The Odds Probability in Favor for m successes out of n is $\frac{m}{m+n}$

The Odds Probability Against for m successes out of n is $\frac{n}{m+n}$

8.7 Probability Distribution of Discrete Random Variable

Probability Distribution: $f(x) = P(X = x)$. The function must satisfy two conditions:

1. $f(x) \geq 0$ for each value in the domain
2. $\Sigma f(x) = 1$

Probability mass function (PMF) must satisfy two conditions:

1. $0 \leq p_X(x) \leq 1$
2. $\Sigma p_X(x) = 1$

8.8 Probability Density of Continuous Random Variable

Probability Density Function: $P(a \leq X \leq B) = \int_a^b f(x)dx$. The function must satisfy two conditions:

For a continuous random variable X, $P(X = a) = 0$

1. $f(x) \geq 0$ for $-\infty < x < \infty$ for each value in the domain
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $\int_a^a f(x)dx = 0 \Rightarrow$probability of any one point is zero

We also have: $P(a \leq X \leq B) = F(b) - F(a)$

Continuous Conditional probability: $f_{X|Y}(x|y) = \frac{f_{X|Y}(x,y)}{f_Y(y)}$

8.9 Cumulative Distribution Function (CDF)

1. Non-decreasing function, since as x grows larger, $P(X \leq x)$ increases
2. $F_x(-\infty) = 0$
3. $F_x(\infty) = 1$

$F(x) = P(X \leq x)$

X is a random variable, x is an algebraic variable

Survival function looks at the complement of the CDF $S(x) = S_X(x) = P(X > x) = 1 - F(x)$

8.10 Discrete Random Variable Probability Mass Function

- Discrete means the range, or possible outcomes, is a finite countable set
- Discrete Variable Probability Mass Function (PMF): $p(x) = p_X(x) = P(X = x)$ $0 \leq p(x) \leq 1$
- $\Sigma p(x) = 1$

8.11 Mixed Distribution

- CDF: $F_X(x) = \alpha_1 F_1(x) + \alpha_2 F_2(x) + \cdots + \alpha_n F_n(x)$ where $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$
- If all X_i are continuous, we have: $f_X(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) + \cdots + \alpha_n f_n(x)$
- Moment Generating Function: $M_X(t) = \alpha_1 M_1(t) + \alpha_2 M_2(t) + \cdots + \alpha_n M_n(x)$

8.12 Expectation

8.12.1 Expected Value of a Random Variable

Discrete Random Variable $E(X) = \sum_{k=x}^{n} x \cdot f(x)$

Continous Random Variable $\int_{-\infty}^{\infty} x \cdot f(x)$

Continuous Random Variable Shortcut: Since the survival function is $S(x) = 1 - F(x)$, we have $E(x) = \int_0^{\infty} S(x)dx$ if the function is non-negative.

Corollary 1, a is a constant: $E(aX) = aE(X)$

Corollary 2, c is a constant: $E(c) = c$

This gives us: $E(aX + b) = aE(x) + b$

Variance: $Var(aX + b) = a^2 \cdot Var(X)$

Variance of a constant c: $Var(c) = 0$

8.12.2 Moments

Discrete Random Variable $E(X^r) = \sum_{x}^{n} x^r \cdot f(x)$

Continuous Random Variable $E(X^r) = \sum_{-\infty}^{\infty} x^r \cdot f(x)dx$

Variance: $Var(X) = E[(X - \mu)^2] = E(X^2) - (E(X))^2$

Standard Deviation: $\sigma = \sqrt{Var(x)}$

8.12.3 Chebyshevs Theorem

The Probability that x is within k standard deviations of the mean is denoted as:

$P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$

8.12.4 Moment Generating Function

Discrete Random Variable: $M_x(t) = E(e^{tx}) = \sum_{x}^{n} e^{tx} \cdot f(x)$

Continuous Random Variable: $M_x(t) = E(e^{tx}) = \sum_{-\infty}^{\infty} e^{tx} \cdot f(x)dx$

Two requirements for a valid moment generating function:

1. $M_x(0) = 1$
2. Variance is greater than 0, so $E(X^2) - (E(X))^2 > 0$

If a and b are constants, then: $M_{aX+b}(t) = e^{bt} \cdot M_X(at)$

If X and Y are independent, then: $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

Coefficient of Variation: $CV(X) = \frac{\sigma}{\mu} = \frac{SD(X)}{E(X)}$

8.13 Special Probability Distributions

8.13.1 Discrete Statistical Distributions

Uniform Measurement	Formula
Mean $\bar{x}$	$\frac{\sum_{i=1}^{n} x_i}{n}$
Median (Odd Number of Values)	$x_{\frac{n+1}{2}}$
Median (Even Number of Values)	$0.5 * (x_{\frac{n}{2}} + x_{\frac{n+1}{2}})$
Mode	Highest Occurring Number
Variance σ^2	$\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$
Standard Deviation σ	$\sqrt{\sigma^2}$
Skewness	$\frac{(\Sigma x_i - \bar{x})^3}{(n-1)\sigma^3}$
Kurtosis	$\frac{(\Sigma x_i - \bar{x})^4}{((\Sigma x_i - \bar{x})^2)^2}$
Standard Error of the Mean	$\frac{\sigma}{\sqrt{n}}$
Average Deviation	$\frac{\lvert x_i - \bar{x} \rvert}{n}$
Range	$x_{imax} - x_{imin}$
Mid-Range	$\frac{x_{imax} - x_{imin}}{2}$
Entropy	$Ln(n)$
Harmonic Mean	$\frac{N}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$
Geometric Mean	$(x_1 * x_2 * \ldots * x_n)^n$

Distribution	Mean	Variance	Probability Formula	Moment Generating Function $M_X(t)$
Bernoulli	np	$p(1-p)$	$p^k q^{n-k}$	$(1-p) + e^{tp}$
Binomial	np	$np(1-p)$	$n!p^k q^{n-k}$	$[1 + np(e^t - 1)]^n$
Negative Binomial	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$	$\frac{(n-1)!p^k q^{n-k}}{(k-1)!(n-k)!}$	
Poisson	λ	λ	$\frac{\lambda^k}{e^\lambda k!}$	$e^{\lambda(e^t - 1)}$
Geometric	$\frac{1}{p}$	$\frac{1-p}{p^2}$	pq^{n-1}	
Hypergeometric	$\frac{nk}{N}$	$\frac{nk(N-k)(N-n)}{N^2(N-1)}$	$\frac{({}_kC_x)*({}_{N-k}C_{n-x})}{{}_NC_n}$	
Multinomial	np_i	$np_i(1-p_i)$	$\frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}$	$(\sum_{i=1}^{k} p_i e^{ti})^n$

The mode is the value that maximizes the mass or density function $\Rightarrow f'(x) = 0$

8.13.2 Quartiles

Quartile	Percent or Formula
First (Lower)	25^{th} percentile
Second (Median)	50^{th} percentile
Third (Upper)	75^{th} percentile
Interquartile Range (IQR)	$75^{th} - 25^{th}$ percentile

$F_X(x) = p$ where p is the percentile in decimal form

Decile goes in 10 percents

8.13.3 Special Probability Densities

Distribution	PDF	Mean	Variance
Uniform	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{1}{12(b-a)^2}$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal Distribution	$\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}} (\frac{x-\mu}{\sigma})^2$	μ	σ^2

Normal Approxiation to the Binomial Distribution is $Z = \frac{X-n\theta}{\sqrt{n\theta(1-\theta)}}$

8.14 Sampling Distributions

8.14.1 Sample Formulas

Sample Mean $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

Sample Variance $S^2 = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}$

8.14.2 Central Limit Theorem

Given a random sample from an infinite population with mean μ and variance σ^2, the limiting distribution is $Z = \frac{\overline{X}-\mu}{\frac{\sigma}{n}}$

8.14.3 Law of Total Expectation

$E(X) = E(E(X|Y))$

Law of Total Variance: $Var(X) = E(Var(X|Y))|Var(E(X|Y))$

8.14.4 Independent Variables

If X and Y are independent, then: $E(XY) = E(X) \cdot E(Y)$

8.14.5 Chi-Square

$\frac{(n-1)S^2}{\sigma^2}$ has a chi-square distribution with n - 1 degrees of freedom.

Chi Squared where O_i is observed values and E_i is expected value

$\chi_c^2 = \Sigma\frac{(O_i-E_i)^2}{E_i}$

8.14.6 t-distribution

If Y has a chi-square distribution with v degrees of freedom, Z has a normal distribution, then the Student t Distribution of $T = \frac{Z}{\sqrt{Y/v}}$ is

$f(t) = \frac{\Gamma\frac{(v+1)}{2}}{\sqrt{\pi v}\Gamma\frac{v}{2}} \cdot (1+\frac{t^2}{v})^{-\frac{v+1}{2}} for - \infty < t < \infty$

8.14.7 F-distribution

F Distribution

When U and V are independent random variables that have chi-square distributions with v_1 and v_2 degrees, of freedom, then:

$F = \frac{\frac{U}{v_1}}{\frac{V}{v_2}}$

is a random variable having an F-distribution

8.15 Decision Theory

8.15.1 Game Theory

Payoff Matrix

PlayerB/PlayerA	a_1	a_2
θ_1	$L(a_1,\theta_1)$	$L(a_2,\theta_1)$
θ_2	$L(a_1,\theta_2)$	$L(a_2,\theta_2)$

where L represents the loss function

8.16 Estimation Theory

$\hat{\Theta}$ is an unbiased estimator of θ if $E(\hat{\Theta}) = \theta$

8.16.1 Maximum Likelihood

$L(\theta) = f(x_1, x_2, \cdots, x_n; \theta)$

8.17 Confidence Intervals

8.17.1 Confidence Interval of the Mean

Confidence Interval for a Large Sample for the Mean $\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

Confidence Interval for a Small Sample for the Mean $\bar{x} - t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\frac{\alpha}{2},n-1} \cdot \frac{s}{\sqrt{n}}$

8.17.2 Confidence Interval of the Difference Between Means

Confidence Interval for a Large Sample for the Difference of Means

$(\bar{x_1} - \bar{x_2}) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x_1} - \bar{x_2}) + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

Confidence Interval for a Small Sample for the Difference of Means

$(\bar{x_1} - \bar{x_2}) - t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} < \mu_1 - \mu_2 < (\bar{x_1} - \bar{x_2}) + t_{\frac{\alpha}{2}, n_1+n_2-2} \cdot s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

8.17.3 Confidence Interval for Estimation of Proportions

Confidence Interval for a Proportion: $\hat{\theta} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}} < \theta < \hat{\theta} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{\theta}(1-\hat{\theta})}{n}}$

8.17.4 Confidence Interval for Difference of Proportions

Confidence Interval for the Difference of Proportions:

$(\hat{\theta}_1 - \hat{\theta}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}} < (\theta_1 - \theta_2) < \hat{\theta}_1 - \hat{\theta}_2 + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{\theta}_1(1-\hat{\theta}_1)}{n_1} + \frac{\hat{\theta}_2(1-\hat{\theta}_2)}{n_2}}$

8.17.5 Confidence Interval of the Variance

Confidence Interval for Variance:

$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}, n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}$

8.18 Hypothesis Testing

8.18.1 Hypothesis Errors

- Type I Error - Reject the null hypothesis when it is true. This probability equals α
- Type II Error - Accept the null hypothesis when it is false. This probability equals β

8.18.2 Neyman-Pearson Lemma

$L_0 = \prod_{i=1}^{n} f(x_i; \theta_0)$ and $L_1 = \prod_{i=1}^{n} f(x_i; \theta_1)$

8.18.3 Power Function of a Test

For H_0 against H_1

$$\pi(\theta) = \begin{cases} \alpha(\theta) \\ 1 - \beta(\theta) \end{cases}$$

8.18.4 Tests Concerning Means

$H_0 : \mu = c$

$H_A : \mu \neq c$

Test Statistic: $Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

Critical Value (CV): Reject H_0 if $z \leq -CV$ or $z \geq CV$

8.18.5 Tests for the Difference of Means

$H_0 : \mu_1 - \mu_2 = 0$

$H_A : \mu_1 - \mu_2 > 0$

Test Statistic

$t = \frac{\bar{x}_1 - \bar{x}_2 - \sigma}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ where $s_p = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

8.18.6 Tests for Proportions

$H_0 : p = c$

$H_A : p \neq c$

Test Statistic

$\hat{p} = \frac{x}{n}$

$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

8.18.7 Proportion Sample Size

If the proportion $\hat{p}$ is known

$n = \frac{z^2 * \hat{p} * (1-\hat{p})}{SE^2}$ where $\hat{p} = \frac{p}{P}$

If the proportion $\hat{p}$ is NOT known

$n = \frac{z^2 * * 0.25}{SE^2}$

8.18.8 Goodness of Fit

Test Statistic is $\chi^2 = \frac{(f_i - e_i)^2}{e_i}$

$\bar{x} = \frac{\sum xf}{n}$

$s^2 = \frac{\sum (x-\bar{x})^2 f}{n-1}$

8.19 Regression and Correlation

Covariance and Regression

$COV(X,Y) = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n}$

$COV(X,Y) = E(XY) - E(X) \cdot E(Y)$

Correlation Coefficient $r = \frac{COV(X,Y)}{s_x s_y}$

$COV(X,Y) = E[(X - \mu_x)(Y - \mu_y)]$

Coefficient of Determination $= r^2$

Least Squares Regression Line $\hat{y} = \alpha + \beta x$

$\beta = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$

8.19.1 Covariance Properties

- $Cov(X,Y) = Cov(Y,X) \Rightarrow$ Symmetry
- $Cov(X,X) = Var(X)$
- $Cov(aX,Y) = a \cdot Cov(X,Y)$
- If a is a constant, then $COV(a,X) = 0$
- If a and b are constants, then $COV(a,b) = 0$
- If a and b are constants, then $COV(aX,bY) = ab \cdot COV(X,Y)$
- If a and b are constants, then $COV(X+a,Y+b) = COV(X,Y)$
- If a and b are constants, then $Var(aX+bY) = a^2 \cdot Var(X) + 2ab \cdot COV(X,Y) + b^2 \cdot Var(Y)$
- If X and Y are independent, then $COV(X,Y) = 0$
- $Cov(X,Y)^2 \leq Var(X) \cdot Var(Y) \Rightarrow$ Cauchy-Schwartz Inequality

8.20 Joint Moment Generating Functions

$M_{X,Y}(s,t) = E(e^{sX+tY})$

8.21 Mean Absolute Percentage Error (MAPE)

1. Take a set of actual values $(a_1, a_2, \cdots, a_n)$
2. Take a set of forecasted values $(f_1, f_2, \cdots, f_n)$
3. For each a_i and f_i, we calculate $\frac{|a_i - f_i|}{|a_i|}$
4. Sum up all the values in Step 3
5. MAPE $= \frac{100\% * Sum}{n}$

8.22 Odds Ratio

Given two events:

- Event X with success probability p_x and failure probability $1 - p_x = q_x$
- Event Y with success probability p_y and failure probability $1 - p_y = q_y$

We have the following grid:

Event	Y = 1	Y = 0
X = 1	p_x	q_x
X = 0	p_y	q_y

We have the following conditional probabilities:

Event	Y = 1	Y = 0
X = 1	$\frac{p_{11}}{(p_{11}+p_{10})}$	$\frac{p_{10}}{(p_{11}+p_{10})}$
X = 1	$\frac{p_{01}}{(p_{01}+p_{00})}$	$\frac{p_{00}}{(p_{01}+p_{00})}$

Our odds ratio is $\frac{p_{11} * p_{00}}{p_{10} * p_{01}}$

8.23 Fishers Exact Test

The Fisher exact test table setup below:

Item	Column 1	Column 2	Row Total
Row 1	a	b	$a+b$
Row 2	c	d	$c+d$
Column Total	$a+c$	$b+d$	$n=(a+b+c+d)$

Probability (p) $= \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{a!b!c!d!n!}$

8.24 Fishers Transformation

Given r, we have: $a = \frac{1+r}{1-r}$

$z = \frac{Ln(a)}{2}$

8.25 Fishers Inverse

Given z, we have: $r = \frac{e^{2z}-1}{e^{2z}+1}$

8.26 Grand Mean

The Grand Mean (GM) is the mean of means.

1. Take (m) number sets
2. Take the mean of each number set n_i
3. Add up all the means: $N = n_1 + n_2 + \cdots + n_i$
4. $GM = \frac{N}{m}$

8.27 Trimmed Mean

Given a number set with (n) components and a trimmed mean percentage (p), we have:

1. $g = \lfloor p * n \rfloor$
2. We remove the bottom (g) entries and the top (g) entries from the original number set to get a trimmed number set (t).
3. Our trimmed number set now has $n - 2g$ entries
4. Calculate the mean of the trimmed number set $\frac{\Sigma t_i}{n-2g}$

8.28 Winsorized Mean

Given a number set with (n) components and a winsorized mean percentage (p), we have:

1. $g = \lfloor p * n \rfloor$
2. We remove the bottom (g) entries and the top (g) entries from the original number set.
3. We replace them with the next closest entries for a new number set w
4. The new number set still has (n) entries
5. Calculate the mean of the new number set $\frac{\Sigma w_i}{n}$

8.29 Missing Average

Given a set of scores $(s_1, s_2, \cdots, s_n)$ and a target average amount (A), the missing value (m) to meet that average is:

$m = n * A - \Sigma s_n$

8.30 Coin Toss Probability

$P(H) = \frac{1}{2} = 0.5$

$P(T) = \frac{1}{2} = 0.5$

8.31 Event Likelihood

An event (e) is a valid probability is $0 \leq P(e) \leq 1$

8.32 Mcnemar Test

The Mcnemar table setup below:

Item	Column 1	Column 2	Row Total
Row 1	a	b	$a + b$
Row 2	c	d	$c + d$
Column Total	$a + c$	$b + d$	$n = (a + b + c + d)$

Set up hypothesis test:

$H_0 : p_b = p_c$

$H_1 : p_b \neq p_c$

Test Statistic: $\chi^2 = \frac{(b-c)^2}{b+c}$

Calculate critical value and if test stat is in rejection region, reject H_0

8.33 Point Estimate and Margin of Error

Given a lower bound l, an upper bound u, and a sample size n, calculate the point estimate (PE) and margin of error (MOE).

$PE = \frac{u+l}{2}$

$MOE = u - PE$

8.34 Analysis of Variance (ANOVA)

8.34.1 One-Way Analysis of Variance (ANOVA)

$SST = SS(Tr) + SSE$

$f = \frac{MS(Tr}{MSE}$

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	f
Treatments	k - 1	SS(Tr)	MS(Tr)	$\frac{MS(Tr)}{MSE}$
Error	k(n - 1)	SSE	MSE	
Total	kn - 1	SST		

8.34.2 Two-Way Analysis of Variance (ANOVA)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	f
Treatments	k - 1	SS(Tr)	MS(Tr)	$\frac{MS(Tr)}{MSE}$
Blocks	n - 1	SSB	MSB	$\frac{MSB}{MSE}$
Error	(n - 1)(k - 1)	SSE	MSE	
Total	kn - 1	SST		

8.35 Nonparametric Test

Sign Test Large Sample

$\mu = n\theta$

$\sigma = \sqrt{n\theta(1-\theta)}$

$Z = \frac{x-\mu}{\sigma}$

8.36 Rule of Succession

Rule of Succession s successes in n independent trials, what is the probability that the next repetition is a success

$P(X_{n+1}|X_1 + ... + X_n = s) = \frac{s+1}{n+2}$ where s is the number of successes and n is the number of trials

9 Finance

Calculators

9.1 Interest and Discount

Interest and Discount Relationships	Variables
$v = \frac{1}{1+i}$	v = discount factor and i = interest rate
$d = \frac{i}{1+i}$	v = discount factor and i = interest rate
$d = iv$	v = discount factor and i = interest rate
$v = 1 - d$	v = discount factor and d = rate of discount
$\sigma = Ln(1+i)$	σ = force of interest and i = interest rate
$i = \frac{d}{1-d}$	i = interest rate and d = rate of discount

9.1.1 Simple and Compound and Continuous Interest

Accumulated Value using Simple Interest $\Rightarrow AV = P(1+it)$

Accumulated Value using Compound Interest $\Rightarrow AV = P(1+i)^t$

Accumulated Value using Continuous Interest $\Rightarrow AV = Pe^{rt}$

where i = interest rate, t = time, e = Euler's constant, and r = rate

9.1.2 Simple and Compound Discount

Accumulated Value using Simple Discount $\Rightarrow AV = \frac{P}{1-dt}$

Accumulated Value using Compound Discount $\Rightarrow AV = \frac{P}{(1-d)^t}$

where d = rate of discount and t = time

9.2 Interest Applications

9.2.1 Rule of 72

The Rule of 72 states that the time (n) for money to double using an interest rate of (i) is $n \approx \frac{0.72}{i}$

Exact Time for Money to double $\Rightarrow n = \frac{Ln(2)}{Ln(1+i)}$

When given an interest rate, make sure it is in percentage form, so 6 percent is 0.06 as a decimal, but we use 6. $\frac{72}{6} = 12$

9.3 Annuities

Annuity Calculation	Formula
Present Value of Annuity Immediate	$PV = Payment * \frac{a-v^n}{i}$
Present Value of Annuity Due	$PV = Payment * \frac{a-v^n}{d}$
Present Value of Continuous Annuity Immediate	$PV = Payment * \frac{a-v^n}{\sigma}$
Accumulated Value of Annuity Immediate	$AV = Payment * \frac{(1+i)^n-1}{i}$
Accumulated Value of Annuity Due	$AV = Payment * \frac{(1+i)^n-1}{d}$
Accumulated Value of Continuous Annuity Immediate	$AV = Payment * \frac{(1+i)^n-1}{\sigma}$
Present Value of Geometric Annuity Immediate	$Payment * \frac{1-\frac{1+k}{1+i}^n}{i-k}$
Accumulated Value of Geometric Annuity Immediate	$Payment * \frac{(1+i)^n-(i+k)^n}{i-k}$

9.4 Perpetuities

Method	Formula	Notes
Present Value of Perpetuity Immediate	$PV = \frac{P}{i}$	P = Payment, i = interest rate, payment due at the end of t
Present Value of Perpetuity Due	$PV = \frac{P}{d}$	P = Payment, i = interest rate, payment due at the beginnin
Present Value of Arithmetic Perpetuity Immediate	$PV = \frac{P_1}{i} + \frac{AP}{i^2}$	P = Payment, i = interest rate, payment due at the end of t
Present Value of Arithmetic Perpetuity Due	$PV = \frac{P_1}{d} + \frac{AP}{d^2}$	P = Payment, A = Arithmetic Amount, i = interest rate, pa

9.5 Yield Rates

9.5.1 Time Weighted Interest

Time Weighted Interest $\frac{MV_1-MV_0+D_1-CF_1}{MV_0}$

9.5.2 Dollar Weighted Interest

Dollar Weighted Interest

$I = \frac{2*(InvestIncome-InvestExpenses)}{/}(Asset + Asset + Income - Expenses)$

9.5.3 Effective Yield

Compounding Period	n
Daily	365
Weekly	52
Semi-monthly	24
Monthly	12
Quarterly	4
Semi-annually	2
Annually	1

Nominal Interest Rate $= i_n$

Effective interest rate $i_e = \frac{i_n}{n}$

Yield Rate $= (1 + i_e)^n - 1$

9.5.4 Nominal Yield

Compounding Period	n
Daily	365
Weekly	52
Semi-monthly	24
Monthly	12
Quarterly	4
Semi-annually	2
Annually	1

$n_r = n * e^{(Ln(EffectiveRate+1)/n)} - 1$

9.5.5 Inflation

The relationship between Nominal Interest, Real Interest, and Inflation is $1 + n = (1 + i)(1 + r)$

9.6 Amortization

9.6.1 Mortgage

The Mortgage formula is $P = \frac{L*APR}{1-(\frac{1}{1+APR})^n}$

The Interest Only Mortgage formula is $P = \frac{L*APR}{12}$

Loan Piece	Symbol	Formula
Discount Factor	v	$\frac{1}{1+APR}$
Interest Paid at time t	I_t	$P(1 - v^{n-t+1})$
Principal Paid at time t	P_t	$P - I_t$
Balance at time t	B_t	$\frac{P(1-v^{n-t})}{APR}$

9.6.2 Loan with Varying Series of Payments

A loan (L) with a series of (n) periodic installments payment at time (t) of R_t:

$L = \sum_{i=1}^{n} v^t R_t$

9.6.3 Sinking Fund

The Sinking Fund Payment = Deposit (D) + Interest (I) where:

$\frac{Loan*i}{(1+i)^n-1}$ and $I = L * i$

9.7 Depreciation Formulas

Depreciation Method	Depreciation at time t D_t	Book Value at time t B_t
Straight Line	$\frac{A-S}{n}$	$(1-\frac{t}{n})*A+S*\frac{t}{n}$
Declining Balance	$dA(1-d)^{t-1}$	$A(1-d)^t$
Double Declining Balance	$\frac{200*B_{t-1}}{n}$	$A-\Sigma B_t$
Sum of the Years Digits	$\frac{(A-S)(N-t+1)}{\Sigma_{x\to 0}n}$	Book Value at time t B_t
Sinking Fund	$\frac{(A-S)(1+j)^{t-1}}{s_n}$	N/A
Units of Output	$\frac{(A-S)(PeriodUnits)}{ProductionUnits}$	N/A

The Capitalized Cost and Periodic Charge: $H = Ai + D + M$ where

Ai is the loss of interest on the original purchase price of the Asset (A)

D is the Depreciation Expense using a Sinking Fund Method

M is the Maintenance Expense

9.8 Interest Rate Approximation Formulas

Method	Formula
Maximum Yield	$\frac{2*YearlyPayments*FinanceCharge}{Loan*(n+1)-FinanceCharge*(n-1)}$
Minimum Yield	$\frac{2*YearlyPayments*FinanceCharge}{Loan*(n+1)+FinanceCharge*(n-1)}$
Constant Ratio	$\frac{2*YearlyPayments*FinanceCharge}{L(n+1)}$
Direct Ratio	$\frac{2*YearlyPayments*FinanceCharge}{L(n+1)+\frac{1}{3}*FinanceCharge*(n-1)}$

9.9 Bonds

9.9.1 Bond Price Methods

- P = Price of the Bond
- F = Par Value of Face Amount
- C = Redemption Value
- r = coupon rate
- Fr = coupon amount
- g = modified coupon rate
- i = yield rate
- n = number of periods from calculation date to redemption date
- K = Present Value
- G = Base Amount = Fr/i

Bond Price Method	Bond Price Formula
Basic	$P = Face * Coupon * \frac{1-v^t}{y} + \frac{R}{1+y^t}$
Premium/Discount	$P = R + (Face * Coupon - R * yield) * \frac{1-v^t}{yield}$
Base	$P = \frac{Face*Coupon}{yield} + (R - \frac{Face*Coupon}{yield})(1 + yield)^t$
Makeham	$P = \frac{R}{(1+yield)^t} + \frac{Face*Coupon}{Yield*R} * (R - \frac{R}{(1+yield)^t})$
Zero Coupon	$P = \frac{F}{(1+yield)^t}$

9.9.2 Bond Yield Methods

Bond Yield Method	Yield Rate Formula
Yield Approximation	$y = \frac{g-\frac{k}{n}}{1+\frac{(k(n+1)}{2n})}$
Bond Salesman	$y = \frac{g-\frac{k}{n}}{1+0.5k}$

9.9.3 Bond Price Items

Method	Flat Price B_{t+k}	Accrued Coupon Fr_k	Market Price = Flat Price - Accrued Coupon
Theoretical	$B_t(1+i)^k$	$Fr[\frac{(1+i)^k-1)}{i}]$	$B_t(1+i)^k - Fr[\frac{(1+i)^k-1)}{i}]$
Practical	$B_t(1+ki)$	kFr	$B_t(1+ki) - kFr$
Semi-Theoretical	$B_t(1+i)^k$	kFr	$B_t(1+i)^k - kFr$

9.9.4 Forward Price

Forward Rate ${}_{t1}f_{t2} = \frac{(1+r_2)^{t_2}}{(1+r_1)^{t_1}}$

Cost of Carry Forward Price $F = (S+s)e^{(r-c)t}$

9.9.5 Method of Equated Time

Given cash flows CF at times (t) and Equivalent Payment EP:

$\bar{t} = \frac{\Sigma CF_t * t}{\Sigma CF_t}$

$v = \frac{1}{1+i}$

$EPv^t = \Sigma CF_t * v^t$

9.9.6 Macaulay Duration and Volatility

Given cash flows CF at times (t) and discount rate (i):

$\bar{d} = \frac{\Sigma CF_t * v^t * t_n}{\Sigma CF_t * v^t}$

$v = \frac{\bar{d}}{1+i}$

9.10 Stocks and Options

9.10.1 Capital Asset Pricing Model

$E[r_k] = r_f + \beta_k(E[r_p] - r_f)$

- r_k = yield rate on a security k
- r_f = Risk Free rate of interest
- r_f = yield rate on the market portfolio
- β_k = risk for security k
- $E[r_p] - r_f$ = risk premium for the market portfolio
- $\beta_k = \frac{cov[r_k, r_p]}{var[r_p]}$

9.10.2 Short Sale Yield

Short Sale Yield $\frac{SalePrice - BuybackPrice + InterestonMargin - Dividends}{MarginRequirement}$

9.10.3 Dividend Discount Model

The present value (PV) of dividends (D) which represent the price (P) which increase by (k) percent with a yield rate of (i) percent is:

Dividend Discount Model $PV = \frac{D}{i-k}$

where D = Initial Dividend Amount, k = dividend increase rate, and i = yield rate

9.10.4 Black-Scholes

Black-Scholes Formula

Call Option Value $C = S * \Phi(d_1) - E * e^{-rt} * \Phi(d_2)$

Put Option Value $P = E * e^{-rt} * (1 - \Phi(d_2)) - S(1 - \Phi(d_1))$

$d_1 = \frac{Ln(S/E) + (r + 0.5\sigma^2) * t}{\sigma\sqrt{t}}$

$d_2 = d_1 - \sigma\sqrt{t}$

9.10.5 Options

Call Options are the right to buy a stock at a certain price with intrinsic value $S - E$

Put Options are the right to sell a stock at a certain price with intrinsic value $E - S$

Put-Call Parity $C + Ke^{-rT} = P + S_0$

Time Value = Option Value - Intrinsic Value

Calls and Puts

$pi = \frac{HighPrice - S}{HighPrice}$

$pl = \frac{LowPrice - S}{LowPrice}$

$p = \frac{r_f - pl}{pi - pl}$

$C = \frac{p * (p^+ - E)}{1 + r_f}$

$P = \frac{p * (E - p^-)}{1 + r_f}$

$\Delta = \frac{E - p^-}{p^+ - p^-}$

Cox Ross Rubenstein

$C = \frac{n! * p^k (1-p)^{n-k} * max(0, u^k d^{n-k} S - E)}{rr^n (k!)(n-k)!}$

$P = \frac{n! * p^k (1-p)^{n-k} * max(0, E - u^k d^{n-k} S)}{rr^n (k!)(n-k)!}$

Treynor Ratio $\Rightarrow TR = \frac{R_f + \beta(R_m - R_f)}{\beta}$

Weighted Average Cost of Capital $WACC = r_D * (1 - T)D + r_E * E$

Volatility Returns $= \frac{Ln(R_1)}{Ln(R_0)}$

9.10.6 Fibonacci Retracements levels

Price 1 = P_1 and Price 2 = P_2

High Price (HP) = Max(P_1, P_2)

Impulse Length (IL) = $P_1 - P_2$

Retracement Percentages (rp) are 23.6%,38.2%,50%,61.8%,100%

Retracements Level = $HP - IL * rp$

9.11 T-Bills

T-Bill price is $P = F - \frac{F*(7*w*y)}{360}$ where F = Face Value, w = weeks, and y = yield percent

9.12 High Low Method

High Cost = c_h, Low Cost = c_l, Production High = p_h, Production Low = p_l

Variable Cost per unit (b) = $\frac{c_h - c_l}{p_h - p_l}$

Total Fixed Cost (TFC) = $c_h b(p_h)$

Total Fixed Cost (TFC) = $c_l b(p_l)$

Cost-Volume Formula (y): $y = TFC + bx$

10 Discrete Math

Discrete Math Calculators

10.1 Truth Tables

Negation Truth Table

P	$\neg P$
T	F
F	T

Logical Disjunction Truth Table

P	Q	P $\vee$ Q
T	T	T
T	F	T
F	T	T
F	F	F

Logical Conjunction Truth Table

P	Q	P $\wedge$ Q
T	T	T
T	F	F
F	T	F
F	F	F

Material Implication Truth Table

P	Q	P $\Rightarrow$ Q
T	T	T
T	F	F
F	T	T
F	F	T

Material Equivalence Truth Table

P	Q	P $\Leftrightarrow$ Q
T	T	T
T	F	F
F	T	F
F	F	T

10.2 Modulus Operations

Modulus Operations⇒ $a \bmod b$ is the remainder left over after a is divided by b

The Quotient Remainder Theorem states that for integer n and positive integer d, $q = \frac{n}{d}$ and $r = n\%d$ then:

$n = dq + r$ where $0 \leq r < d$

Congruence Modulo N is determined if $a \equiv b \bmod n$, then $n|a - b$

10.3 Multifactorials and Subfactorials

Multifactorial $n!^m = n * (n - m) * \cdots * 1$

Subfactorial or Derangement $!n = [\frac{n!}{e} + 0.5]$

10.4 Partitions

Unordered Partitions $k = \frac{n}{m}$ and $a = \frac{n!}{(k!)(m!^k)}$

Ordered Partitions $k = \frac{n}{m}$ and $a = \frac{n!}{m!^k}$

Cross Partitions

Cross Partitions are the intersection of all items of a partition with subsets of other partitions.

$[1, 2, 3, 4, 5, 6][1, 4, 2, 3, 5, 6][2, 84, 7, 9, 3, 6]$

$1, 2, 3, 2, 3, 4, 5, 6, 4, 6, 4, 2, 3, 6$

10.5 Partitioned Intervals

1. Form each subinterval $S_n = [x_n, x_{n+1}]$
2. Calculate each subinterval length $\Delta_n = x_{n+1} - x_n$
3. Calculate the norm (mesh) which is the maximum value of all subinterval lengths

10.6 Interpolation

Linear Interpolation formula: $y_a + \frac{(y_b - y_a)(x - x_a)}{x_b - x_a}$

Nearest Neighbor Interpolation formula for a point p: $\sum_{k=0}^{n} |x_k - p|$, and then find the lowest difference d and use $f(d)$

10.7 Unique Word Arrangements

A word with M letters in it has the following unique word arrangements:

$\frac{M}{N_1 N_2 \cdots N_M}$

where N_i is the number of duplicates of a particular letter

10.8 Primitive Root

If p is prime, then b is a primitive root for p if the powers of b include all of the residue classes mod p. Use b = 3 and p = 7 as an example

n	n - 1	b^{n-1}	b^{n-1} mod p
1	0	$3^0 = 1$	$3^0 \bmod 7 = 1$
2	1	$3^1 = 3$	$3^1 \bmod 7 = 3$
3	2	$3^2 = 9$	$3^2 \bmod 7 = 2$
4	3	$3^3 = 27$	$3^3 \bmod 7 = 6$
5	4	$3^4 = 81$	$3^4 \bmod 7 = 4$
6	5	$3^5 = 243$	$3^5 \bmod 7 = 5$

Since we achieved all values from 1 to 6 in our residue results, then 3 is a primitive root of 7.

10.9 Linear Congruence

$ax \equiv c(\text{mod b})$

1. Determine the GCF(a, b)
2. If the GCF is 1, then there will be 1 solution mod c
3. Set up the Diophantine Equation $ax + by = c$
4. Solve the Linear Congruence using Euclid Algorithm

10.10 Euclids Algorithm and Diophantine Equations

Diophantine Equation $ax + by = c$, then $d = gcd(a, b)$

Algorithm 10.10.1 Euclidean Algorithm

```
procedure EUCLID(a, b)
    r := a mod b
    while b ≠ 0 do
        a := b
        b := r
        r := a mod b
    end while
    return a                                ▷ The gcd is the positive integer a
end procedure
```

Algorithm 10.10.2 Extended Euclidean Algorithm

```
procedure EXTEUCLID(A, B)
    a := A, b := b, s := 1, t := 0, u := 0, v := 1
    while b ≠ 0 do
        r := a mod b, q := a ÷ b
        a := b, b := r
        newu := s − uq, newv := t − vq
        s := u, t := v
        u := newu, v := newv
    end while
    return a                                ▷ The gcd is the positive integer a
end procedure
```

Algorithm 10.11.1 Collatz Conjecture Algorithm

```
procedure Collatz(N)
    n := N
    while n ≠ 1 do
        if i mod 2 = 0 then
            n := n/2                          ▷ Even number
        else
            n := 3n − 1                       ▷ Odd Number
        end if
    end while
    return n                                  ▷ Positive Integer n
end procedure
```

10.11 Collatz Conjecture

10.12 Prime Number Algorithms

The totient of a number n is denoted as φ

1. List all factors for n
2. for $k = 1$, $k{+}{+}$, while $k < n$ List Factors for k
3. Pick up all numbers who share no factors with n other than 1

The Sieve of Eratosthenes is used to find prime numbers before a number n

1. Find the first number m less than n where $m^2 > n$
2. Subtract 1 from m: $m = m - 1$
3. List all numbers from 2 to n and call this list L
4. for $k = 2$, $k{+}{+}$, while $k <= m$ Remove all multiples of k, modify L.
5. When m = k, after List is modified, the List will be $\pi(n) = L$

Fermats Little Theorem states that if p is a prime number, than for any integer a, ap - a is an integer multiple of p

$ap - 1 \equiv 1(\text{mod} p)$

10.13 Other Number Algorithms

The Lagrange Four Square Theorem states that any natural number can be expressed as:

$p = a^2 + b^2 + c^2 + d^2$

10.14 Number Properties

Number Type	Description/Formula	First Four Examples
Perfect	Sum of Divisors = Number	6, 28, 496, 8128
Abundant	Sum of Divisors > Number	12, 18, 20, 24
Deficient	Sum of Divisors < Number	1, 2, 3, 4
Evil	Even Number of One's in Binary Expansion	3, 5, 6, 9
Odious	Odd Number of One's in Binary Expansion	1, 2, 4, 7
Triangular	each row has 1 more item in a triangular form	1, 3, 6, 10
Automorphic (Curious)	decimal expansion of n^2 ends with n	1, 5, 6, 25
Undulating	alternating digits in the form $abab$	101, 111, 121, 131,
Square	has an integer square root	1, 4, 9, 16
Cube	has an integer cube root	1, 8, 27, 64
Square	has an integer square root	1, 4, 9, 16
Triangular	satisfies $\frac{n(n+1)}{2}$	1, 3, 6, 10
Rectangular	satisfies $n(n+1)$	2, 6, 12, 20
Palindrome	read the same forward and backward	11, 22, 33, 44
Palindromic Prime	Palindrome and Prime	101, 131, 151, 181
Repunit	Every digit equal to 1	11, 111, 1111, 11111
Apocalyptic Power	2^n contains 666	157, 192, 218, 220
Pentagonal	satisfies $\frac{n(3n-1)}{2}$	1, 5, 12, 22
Hexagonal	satisfies $n(2n-1)$	1, 6, 15, 28
Heptagonal	satisfies $\frac{n(5n-3)}{2}$	1, 7, 18, 34
Octagonal	satisfies $n(3n-2)$	1, 8, 21, 40
Nonagonal	satisfies $\frac{n(7n-5)}{2}$	1, 9, 24, 46
Tetrahedral (Pyramidal)	satisfies $\frac{n(n+1)(n+2)}{6}$	1, 4, 10, 20
Narcissistic (Plus Perfect)	m digit number equal to the square sum of it's m-th powers of its digits	1, 2, 3, 4
Catalan	$C_n = \frac{2n!}{n!(n+1)!}$	1, 2, 5, 14,

11 Linear Algebra

LINEAR ALGEBRA Calculators

11.1 Matrix Operations

Structure is a_{ij} where i = row and j = column

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Scalar Multiplication cA

$$cA = \begin{vmatrix} ca_{11} & ca_{12} & ca_{13} \\ ca_{21} & ca_{22} & ca_{23} \\ ca_{31} & ca_{32} & ca_{33} \end{vmatrix}$$

Matrix Addition - Add respective entries of A and B: $|A| + |B|$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{vmatrix}$$

Matrix Subtraction - Subtract respective entries of A and B: $|A| - |B|$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} - \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11}-b_{11} & a_{12}-b_{12} & a_{13}-b_{13} \\ a_{21}-b_{21} & a_{22}-b_{22} & a_{23}-b_{23} \\ a_{31}-b_{31} & a_{32}-b_{32} & a_{33}-b_{33} \end{vmatrix}$$

Matrix Multiplication: $|A| * |B|$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} * \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31} & a_{11}b_{12}+a_{12}b_{22}+a_{13}b_{32} & a_{11}b_{13}+a_{12}b_{23}+a_{13}b_{33} \\ a_{21}b_{11}+a_{22}b_{21}+a_{23}b_{31} & a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32} & a_{21}b_{13}+a_{22}b_{23}+a_{23}b_{33} \\ a_{31}b_{11}+a_{32}b_{21}+a_{33}b_{31} & a_{31}b_{12}+a_{32}b_{22}+a_{33}b_{32} & a_{31}b_{13}+a_{32}b_{23}+a_{33}b_{33} \end{vmatrix}$$

11.2 Matrix Properties

Determinant

Inverse

Adjoint

11.3 Cross Product in R3

$$\begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

$X \cdot Y = (x_2y_3 - x_3y_2)i + (x_3y_1 - x_1y_3)j + (x_1y_2 - x_2y_1)k$

$$A = \begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix} B = \begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix} C = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix} D = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & z_3 & z_3 \end{vmatrix}$$

$|A| = y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2)$

$|B| = z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2)$

$|C| = x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$

$|D| = x_1(y_2z_3 - y_3z_2) + x_2(y_3z_1 - y_1z_3) + x_3(y_1z_2 - y_2z_1)$

11.4 Digraph

Digraph

11.5 Equation of a Plane

The standard equation for a plane is: $Ax + By + Cz + D = 0$

Given three points in space $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$, the equation of the plane through these points is given by the following determinants:

11.6 Plane and Parametric Equations in R3

Given a vector $A = \begin{vmatrix} a \\ b \\ c \end{vmatrix}$ and a point (x_0, y_0, z_0)

The plane equation passing through (x_0, y_0, z_0) and perpendicular to A is denoted as

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

This is a parametric equation of L, which can be written in terms of the components as:

$x = x_0 + tu$

$y = y_0 + tv \;\; (-\infty < t < \infty)$

$z = z_0 + tw$

11.7 Vectors

Given two vectors $A = \begin{vmatrix} a_1 \\ a_2 \\ a_3 \end{vmatrix} B = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix}$

Length (Magnitude) $||A|| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

$A - B = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

$A \cdot B = a_1b_1 + a_2b_2 + a_3b_3$

Distance: $\overline{AB} = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

For $\angle\theta$ between A and B, $cos(\theta) = \frac{||A|| \cdot ||B||}{||A|| * ||B||}$

Unit Vector $\frac{U}{||A||}$

Vectors are perpendicular if $|A| \cdot |B| = 0$

Vectors are parallel if $|A| \cdot |B| = ||A|| * ||B||$

Cauchy-Schwartz $|A \cdot B| \leq ||A|| * ||B||$

Orthogonal Projection $proj_B A = \frac{(A \cdot B) * B}{||B||^2}$

11.8 Markov Chain

The Markov Chain procedure with Transition Matrix T and initial state vector $P^0 \Rightarrow P^{(n)} = TP^{(n-1)}$

12 Set Theory

Set Theory Calculators

12.1 Roster Notation Items

Roster Notation	Symbol	Action
Greater than	$>$	Do not include the last number on the right
Greater than or equal to	$\geq$	Include the last number on the right
Less than	$<$	Do not include the last number on the left
Less than or equal to	$\leq$	Include the last number on the left

The set of all natural numbers less than 8: $\{1, 2, 3, 4, 5, 6, 7\}$

The set of all natural numbers less than or equal to 8: $\{1, 2, 3, 4, 5, 6, 7, 8\}$

The set of all integers less than 5: $\{-\infty, ..., 0, 1, 2, 3, 4\}$

The set of all integers greater than or equal to 5: $\{5, 6, 7, ..., \infty\}$

12.2 Set Notation Items

Set Notation Items	Formula
Union (All Elements in A and B)	$A \cup B$
Intersection (All Elements in A and B	$A \cap B$
Set Difference for A (All Elements in A not in B	$A - A \cap B$
Set Difference for B (All Elements in B not in A	$B - A \cap B$
Symmetric Difference (All Elements in A not in B plus All elements in B not in A	$A \Delta B = (A - B) \cup (B - A)$
Concatenation (All Elements in A and B)	$A \cdot B$
Cardinality (Number of Elements in A)	$\|A\|$
Cardinality (Number of Elements in B)	$\|B\|$
Cartesian Product (All Orderd Pairs in A and B)	AxB
Power Set (All Subsets of A including)	$P(A)$
Jaccard Index $J(A, B)$	$\frac{\|A \cap B\|}{\|A \cup B\|}$
Jaccard Distance $J_\sigma(A, B)$	$1 - J(A, B)$
Dice Coefficient s	$\frac{\|A \cap B\|}{\|A\|+\|B\|}$

12.3 Power Sets

Power Set is the set of all subsets including the empty set.

$S = \{a, b, c, d\}$

When S contains n items, the Power Set P should contain 2^n items

The power set $P = \{\emptyset, a, b, c, d, (a, b), (a, c), (a, d), (b, c), (b, d), (c, d), (a, b, c), (a, b, d), (a, c, d), (b, c, d), (a, b, c, d)\}$

12.4 Partitions of a Set

The number of partitions of a set is the Bell Number $B_{n+1} = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} B_k$

13 Economics

ECONOMICS Calculators

13.1 Formulas

Budget Line Equation	$I = Q_x P_x + Q_y P_y$
Net Exports (NX)	$NX = E - I$
Gross Domestic Product	$GDP = C + I + G + (E - I)$
Gross Domestic Product Deflator	$GDPDeflator = \frac{GDP_{nominal} * 100}{GDP_{real}}$
Equation of Exchange	$MV = PQ$
Money Multiplier	$\frac{1}{RR}$
Gross Profit	$GrossProfit = Revenue - Cost$
Gross Profit Margin	$\frac{GrossProfit}{Revenue}$
Net Profit	$NetProfit = GrossProfit(1 - TaxRate)$
Net Profit Margin	$\frac{NetProfit}{Revenue}$
Sectoral Balance	$PS - PI = GS - T + E - I$
Cost Utility Ratio	$\frac{Cost}{Utility}$
Total Revenue (TR)	$TR = Cost * Quantity$
Herfindahl Index (H)	Σs_i^2
Normalized Herfindahl Index H*	$H^* = \frac{H - 1/N}{1 - 1/N}$

14 Chemistry

14.1 Laws

P = Pressure, V = Volume, T = Temperature

Boyle's Law $\Rightarrow P_1V_1 = P_2V_2$

Charles Law $\Rightarrow V_1T_2 = V_2T_1$

Combined Gas Law $\Rightarrow P_1V_1T_2 = P_2V_2T_1$

Pressure Law $\Rightarrow P_1T_2 = P_2T_1$

14.2 Density

Density $\Rightarrow D = MV$ where M = Mass and V = Volume

15 Biology

15.1 Punnett Square Items

15.1.1 Punnett Square

Punnett Square where A = Dominant Gene and a = Recessive Gene

	A	a
A	AA	Aa
a	Aa	aa

15.1.2 Hardy Weinberg

Hardy Weinberg where A = Dominant Gene and a = Recessive Gene with probabilities p and $q = 1 - p$

	A (p)	a (q)
A (p)	AA (p^2)	Aa (pq)
a (q)	Aa (pq)	aa (q^2)

16 Actuarial Science

16.1 Population and Mortality Factors

Population Formula	$d_x = l_x - l_{x+1}$
Survival Formula	$p_x = \frac{l_{x+1}}{l_x}$
Mortality Formula	$q_x = 1 - p_x$

16.2 Hazard Rate Function

Also called the force of mortality: $\lambda(x) = \frac{f_X(x)}{S_X(x)}$

Density divided by Survival

16.3 Social Security and Covered Compensation

Social Security Calculation

Averaged Indexed Monthly Earnings $AIME = \frac{35yearSalaryHistorywithIndexing}{35*12}$

$Tier1 = 0.9 * min(AIME, BendPoint1)$

$Tier2 = 0.32 * Maximum(Minimum(BendPoint2, AIME) - BendPoint1, 0)$

$Tier3 = 0.15 * Max(0, AIME - BendPoint2)$

Primary Social Security Benefit $PSSB = Tier1 + Tier2 + Tier3$

Reduction Factor 1: $\frac{5}{9} * 0.01 * Min(36, MonthsRetiringbeforeSSNRA)$

Reduction Factor 2: $\frac{5}{9} * 0.01 * Maximum(MonthsRetiringbeforeSSNRA - 36, 0)$

Early Reduction Factor: $ERF = RED1 + RED2$

Reduced Social Security if taken before Primary Social Security Age:

$PSSB * ERF$

Covered Compensation

- Take birth year + Social Security Normal Retirement Age
- Add up the 35 years of compensation history
- Divide this by 35 to get your covered compensation

17 Engineering

ENGINEERING Calculators

17.1 Bending Beams Displacement

Bending Beams Displacement (dual Support) $\Rightarrow d = \frac{FI^3}{4yab^3}$

Bending Beams Displacement (Single Support) $\Rightarrow d = \frac{FI^3}{yab^3}$

17.2 Direct Current Ohms Law

Formulas

$P = V * I$

$R = \frac{V}{I}$

$P = I^2 R$

$I = \frac{V}{R}$

$V = \sqrt{P} * R$

I = Current(Amps), V = (volts), R = Resistance(ohms), P = Power(watts)

17.3 Sine Wave

Formulas

$AverageValue = 0.637 * PeakValue$

$RMS = 0.707 * PeakValue$

$RMS = 1.11 * AverageValue$

$PeakValue = 1.57 * AverageValue$

$PeakValue = 1.414 * RMS$

$Average = 0.9 * RMS$

17.4 Young's Modulus

Youngs Modulus $\Rightarrow Y = \frac{n}{m^2}$

17.5 Static Determinancy and Stability

Static Determinancy and Stability $\Rightarrow 2j = m + 3$

18 Capital Budgeting

Calculators

18.1 Average Returns

Average Return $= \frac{\sum_{i=1}^{n} CF}{n}$

Average Rate of Return $= \frac{AR}{I}$

18.2 Incremental Cash Flow

$ICF = CashInflowsCashOutflows(InflowsOutflowsDepreciationExpense) * TaxRate$

$Taxes = (InflowsOutflowsDepreciationExpense) * TaxRate$

18.3 Net Present Value and IRR and Profitability Index

$PV_t = \frac{CF_t}{(1+i)^t}$

$NPV = \sum_{t=0}^{n} PV_t$

18.4 Modified Internal Rate of Return (MIRR)

$MIRR = \sqrt[n]{\frac{FVofPositiveCF}{PVofNegativeCF}}$

18.5 Equivalent Annual Cost

$EAC = DiscountedInvestment + MaintenanceCost$

19 Physics

PHYSICS

$Acceleration = \frac{v-v_0}{t}$

$AngularMomentum = L = M * V * R$

$CentripetalAcceleration = \frac{v^2}{r}$

Frequency and Wavelength $f = \frac{c}{\lambda}$

Gravitational Force $F = \frac{G*m1*m2}{d^2}$

Kinematic Equations

$d = vi_t + \frac{1}{2}at_2$

$v_f^2 = v_i^2 + 2ad$

$v_f = v_i + at$

$d = \frac{1}{2}(v_i + v_f)t$

Kinetic Energy $KE = \frac{1}{2}MV^2$

Lever Systems Formula $F_1x = F_2(d - x)$

Little's Law $WIP = CT * TH$

$MomentofInertia = Mass * Length^2$

Parallel Resistors $\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\ldots+\frac{1}{R_n}}$

$Work = F * D$

20 Accounting

ACCOUNTING Calculators

20.1 Balance Sheet

Working Capital = Current Assets - Current Liabilities

Current Ratio = $\frac{CurrentAssets}{CurrentLiabilities}$

$QuickAssets = CurrentAssets - Inventory$

Quick / Acid Test / Current Ratio = $\frac{QuickAssets}{CurrentLiabilities}$

Receivable Turnover

20.2 Methods

Installment Sales Method of Accounting

$GrossProfit = \frac{SalesPrice}{Cost}$ and $CostPercentage = 1 - GrossProfitPercentage$

Chain Discount $Pd_1d_2 \cdots d_n = P(1 - d_1)(1 - d_2 \cdots (1 - d_n)$

Percentage of Completion ⇒ Gross Profit to Date = Percent Complete x Profit Amount

Vendor Discount a/b net c ⇒ $\frac{a}{100-a} * \frac{360}{c-b}$

20.3 Inventory

FIFO and LIFO

Average Inventory = $\frac{I_0+I_1}{2}$

Inventory Turnover Ratio = $\frac{COGS}{AI}$

21 Health and Well Being

Health Calculators

Basal Metabolic Rate	$BMR = 66 + 13.7xweightinpounds + 5xheightininches - 6.8ageinyears$
Body Mass Index	$BMI = \frac{WeightinKilos}{HeightInMeters^2}$
Cholesterol	$Cholesterol = HDL + LDL + \frac{1}{5}Triglycerides$

22 Word Problems

Word Problem Calculators

2 number word problems 2 numbers a and b have a sum of s and a product p.

$a + b = s$ and $ab = p$

Consecutive Integer Word Problems 2 consecutive integers have a sum of s.

Number 1 = n, Number 2 = n + 1, so $n + (n + 1) = s$

Distance Problems $d = rt$

Inclusive Number Word Problem Type	Formula
Average of all inclusive numbers A to B	$\frac{A+B}{2}$
Count of all inclusive numbers A to B	$B - A + 1$
Sum of all inclusive numbers A to B	$\frac{(A+B)(B-A+1)}{2}$

Number Type	Sum of the First Formula	First Five Numbers
Whole Numbers	$S_n = \frac{n(n-1)}{2}$	0, 1, 2, 3, 4
Natural Numbers	$S_n = \frac{n(n+1)}{2}$	1, 2, 3, 4, 5
Even Numbers	$S_n = n(n+1)$	2, 4, 6, 8, 10
Odd Numbers	$S_n = n^2$	1, 3, 5, 7, 9
Square Numbers	$S_n = \frac{n(n+1)(2n+1)}{6}$	1, 4, 9, 16, 25
Cube Numbers	$S_n = \frac{n^2(n+1)^2}{4}$	1, 8, 27, 64, 125
Fourth Power Numbers	$S_n = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$	1, 16, 81, 256, 625

22.1 Population Growth

Given an initial population amount of P_0, a percentage growth of g, and a time of t, we have the following population growth at time t:

$P(t) = P_0(1 + g)^t$

23 Conversions

23.1 Coin Conversions

1 dollar	100 pennies
1 dollar	20 nickels
1 dollar	10 dimes
1 dollar	4 quarters
1 dollar	2 half-dollars
1 half-dollar	50 pennies
1 half-dollar	10 nickels
1 half-dollar	5 dimes
1 half-dollar	2 quarters
1 quarter	25 pennies
1 quarter	5 nickels
1 dime	10 pennies
1 dime	2 nickels
1 nickel	5 pennies

23.2 Temperature Conversions

$F = (1.8C) + 32$

$K = (C + 273.15)$

$R = \frac{9(C+273.15)}{5}$

$N = 0.3333C$

$Reaumur = 0.8C$

$C = \frac{9}{5}(F - 32)$

$K = \frac{5}{9}(F + 459.67)$

$R = F + 459.67$

$N = \frac{11}{60}(F32)$

$R = \frac{4}{9}(F32)$

$C = K273.15$

$F = \frac{9}{5}K - 459.67$

$R = \frac{9}{5}K$

$N = 0.33(F273.15)$

$Reaumur = 0.8(K273.15)$

23.3 Roman Numerals

Roman Numeral Letter	Number Value
I	1
II	2
III	3
IV	4
V	5
X	10
L	50
C	100
D	500
M	1,000

To convert a number to Roman Numeral notation, use these steps:

1. Find the highest number in the table that is equal to or lower than your number.
2. Start with that letter
3. Take your number and subtract the table number This is your new number
4. Repeat steps 1-3 until you reach zero as your target number

In Excel, you can use the ROMAN function to convert numbers to Roman Numerals. Example: =ROMAN(50)

23.4 ROT 13

Original Letter	Alphabet Position	Alphabet Position Mod 13	ROT 13 Letter
A	1	1 + 13 = 14	N
B	2	2 + 13 = 15	O
C	3	3 + 13 = 16	P
D	4	4 + 13 = 17	Q
E	5	5 + 13 = 18	R
F	6	6 + 13 = 19	S
G	7	7 + 13 = 20	T
H	8	8 + 13 = 21	U
I	9	9 + 13 = 22	V
J	10	10 + 13 = 23	W
K	11	11 + 13 = 24	X
L	12	12 + 13 = 25	Y
M	13	13 + 13 = 26	Z
N	14	14 + 13 = 27 (Reset portion greater than 26 27 - 26 = 1)	A
O	15	15 + 13 = 28 (Reset portion greater than 26 28 - 26 = 2)	B
P	16	16 + 13 = 29 (Reset portion greater than 26 29 - 26 = 3)	C
Q	17	17 + 13 = 30 (Reset portion greater than 26 30 - 26 = 4)	D
R	18	18 + 13 = 31 (Reset portion greater than 26 31 - 26 = 5)	E
S	19	19 + 13 = 32 (Reset portion greater than 26 32 - 26 = 6)	F
T	20	20 + 13 = 33 (Reset portion greater than 26 33 - 26 = 7)	G
U	21	21 + 13 = 34 (Reset portion greater than 26 34 - 26 = 8)	H
V	22	22 + 13 = 35 (Reset portion greater than 26 35 - 26 = 9)	I
W	23	23 + 13 = 36 (Reset portion greater than 26 36 - 26 = 10)	J
X	24	24 + 13 = 37 (Reset portion greater than 26 37 - 26 = 11)	K
Y	25	25 + 13 = 38 (Reset portion greater than 26 38 - 26 = 12)	L
Z	26	26 + 13 = 39 (Reset portion greater than 26 39 - 26 = 13)	M

Algorithm 23.4.1 Rot13 Algorithm

```
procedure Rot13(word)
    nlen = word.length
    phrase = null
    for i := 1, i ≤ n, i++ do                         ▷ Start at the first letter and go to the end
        letter = n mod 26                              ▷ A=1 and Z=26, anything over 26 is reduced mod 26
        phrase := phrase + LetterNumber(letter)
    end for
    return phrase                                      ▷ Return the numeric phrase
end procedure
```

23.5 Area Conversions

Letter	Number Value
1 hectare	2.4711 acres
1 square mile	640 acres
1 square kilometer	247.105 acres
1 square mile	258.998811 hectares
1 square kilometer	100 hectares
1 acre	6272640 square inchs
1 hectare	15500031 square inchs
1 square foot	144 square inchs
1 square yard	1296 square inchs
1 square mile	4014489600 square inchs
1 square meter	1550.0031 square inchs
1 square kilometer	1550003100 square inchs
1 acre	43560 square foots
1 hectare	107639.104 square foots
1 square yard	9 square foots
1 square mile	27878400 square foots
1 square meter	10.7639 square foots
1 square kilometer	10763910.4 square foots
1 acre	4840 square yards
1 hectare	11959.9005 square yards
1 square mile	3097600 square yards
1 square meter	1.19599005 square yards
1 square kilometer	1195950.05 square yards
1 acre	4046856420 square millimeters
1 hectare	10000000000 square millimeters
1 square inch	645.16 square millimeters
1 square foot	92903.04 square millimeters
1 square yard	836127.36 square millimeters
1 square mile	2589988110336 square millimeters
1 square meter	1000000 square millimeters
1 square kilometer	1000000000000 square millimeters
1 acre	4046.86 square meters
1 hectare	10000 square meters
1 square mile	2589988.11 square meters

23.6 Computer Storage Conversions

1 kilobyte	1024 bytes
1 megabyte	1048576 bytes
1 gigabyte	1073741824 bytes
1 terabyte	1099511627776 bytes
1 petabyte	1125899906842620 bytes
1 exabyte	1152921504606840000 bytes
1 megabyte	1024 kilobytes
1 gigabyte	1048576 kilobytes
1 terabyte	1073741824 kilobytes
1 petabyte	1099511627776 kilobytes
1 exabyte	1125899906842620 kilobytes
1 gigabyte	1024 megabytes
1 terabyte	1048576 megabytes
1 petabyte	1073741824 megabytes
1 exabyte	1099511627776 megabytes
1 terabyte	1024 gigabytes
1 petabyte	1048576 gigabytes
1 exabyte	1073741824 gigabytes
1 petabyte	1024 terabytes
1 exabyte	1048576 terabytes
1 exabyte	1024 petabytes

23.7 Liquid Conversions

1 ounce	29.5735296 milliliters
1 ounce	6 teaspoons
1 ounce	2 tablespoons
1 cup	8 ounces
1 cup	240 milliliters
1 cup	48 teaspoons
1 cup	16 tablespoons
1 liter	1000 milliliters
1 teaspoon	5 milliliters
1 tablespoon	15 milliliters
1 quart	946.352946 milliliters
1 gallon	3785.41178 milliliters
1 pint	473.176473 milliliters
1 gallon	3.78541178 liters
1 liter	202.884136 teaspoons
1 tablespoon	3 teaspoons
1 quart	192 teaspoons
1 gallon	768 teaspoons
1 pint	96 teaspoons
1 liter	67.6280454 tablespoons
1 quart	64 tablespoons
1 gallon	256 tablespoons
1 pint	32 tablespoons
1 liter	4.22675284 cups
1 quart	4 cups
1 gallon	16 cups
1 pint	2 cups
1 liter	1.05668821 quarts
1 gallon	4 quarts
1 liter	2.11337642 pints
1 quart	2 pints
1 gallon	8 pints
1 pint	16 ounces
1 liter	33.8140227 ounces
1 quart	32 ounces

23.8 Speed Conversions

1 mph	1.47 ft/ss
1 m/sec	3.281 ft/ss
1 mph	1.61 km/hs
1 ft/s	1.097 km/hs
1 m/sec	3.6 km/hs
1 mph	2.236 m/secs

23.9 Linear Conversions

1 inch	25.4 millimeters
1 inch	2.54 centimeters
1 inch	25,400 micrometers
1 foot	12 inches
1 foot	304.8 millimeters
1 foot	30.48 centimeters
1 foot	304,800 micrometers
1 yard	36 inches
1 yard	3 feet
1 yard	914.4 millimeters
1 yard	91.44 centimeters
1 yard	914,400 micrometers
1 mile	63,360 inches
1 mile	5,280 feet
1 mile	1,760 yards
1 mile	1,609,344 millimeters
1 mile	160,934.4 centimeters
1 mile	1,609.344 meters
1 mile	1.609344 kilometers
1 mile	8 furlongs
1 mile	1,609,344,000 micrometers
1 millimeter	1,000 micrometers
1 centimeter	10 millimeters
1 centimeter	10,000 micrometers
1 meter	39.3700787 inches
1 meter	3.2808399 feet
1 meter	1.0936133 yards
1 meter	1,000 millimeters
1 meter	100 centimeters
1 meter	1,000,000 micrometers
1 kilometer	39,370.0787 inches
1 kilometer	3,280.8399 feet
1 kilometer	1,093.6133 yards
1 kilometer	1,000 meters
1 kilometer	1,000,000 millimeters

23.10 RGB HEX Conversions

RGB to HEX using (R, G, B) as a numerical entry

$R1 = \frac{R - R \bmod 16}{16}$ and pick off the number letter for 0123456789ABCDEF

$R2 = R \bmod 16$ and pick off the number letter for 0123456789ABCDEF

$G1 = \frac{G - G \bmod 16}{16}$ and pick off the number letter for 0123456789ABCDEF

$G2 = G \bmod 16$ and pick off the number letter for 0123456789ABCDEF

$B1 = \frac{B - B \bmod 16}{16}$ and pick off the number letter for 0123456789ABCDEF

$B2 = B \bmod 16$ and pick off the number letter for 0123456789ABCDEF

Hue $\Rightarrow h = \frac{60^\circ(G-B)}{R-B}$

23.11 12 and 24 Hour Clock Conversions

24 hour to 12 hour conversions: Any PM hours get switched to mod 13.

23.12 Time Conversions

1 second	1000 milliseconds
1 minute	60000 milliseconds
1 hour	3600000 milliseconds
1 day	86400000 milliseconds
1 week	604800000 milliseconds
1 fortnight	8467200000 milliseconds
1 month	2592000000 milliseconds
1 quarter	7776000000 milliseconds
1 year	31104000000 milliseconds
1 decade	311040000000 milliseconds
1 century	3110400000000 milliseconds
1 millenium	31104000000000 milliseconds
1 millisecond	1000000 nanoseconds
1 millisecond	1000 microseconds
1 minute	60 seconds
1 hour	3600 seconds
1 day	86400 seconds
1 week	604800 seconds
1 fortnight	8467200 seconds
1 month	2592000 seconds
1 quarter	7776000 seconds
1 year	31104000 seconds
1 decade	311040000 seconds
1 century	3110400000 seconds
1 millenium	31104000000 seconds
1 second	1000000000 nanoseconds
1 hour	60 minutes
1 day	1440 minutes
1 week	10080 minutes
1 fortnight	141120 minutes
1 month	43200 minutes
1 quarter	129600 minutes
1 year	518400 minutes
1 decade	5184000 minutes
1 century	51840000 minutes

23.13 Unit Conversions

1 pair	2 units
1 half-dozen	6 units
1 half-dozen	3 pairs
1 dozen	12 units
1 dozen	6 pairs
1 dozen	2 half-dozens
1 bakers-dozen	13 units
1 gross	144 units
1 gross	72 pairs
1 gross	24 half-dozens
1 gross	12 dozens

23.14 Weight Conversions

1 pound	16 ounces
1 kilogram	35.2739619 ounces
1 stone	224 ounces
1 ton	32000 ounces
1 kilogram	2.20462262 pounds
1 stone	14 pounds
1 ton	2000 pounds
1 ounce	28349.5231 milligrams
1 pound	453592.37 milligrams
1 gram	1000 milligrams
1 kilogram	1000000 milligrams
1 stone	6350293.8 milligrams
1 ton	907184740 milligrams
1 centigram	10 milligrams
1 ounce	28.3495231 grams
1 pound	453.59237 grams
1 kilogram	1000 grams
1 stone	6350.29318 grams
1 ton	907184.74 grams
1 stone	6.35029318 kilograms
1 ton	907.18474 kilograms
1 ton	142.857143 stones
1 ounce	2834.951826 centigrams
1 pound	45359.22922 centigrams
1 gram	100 centigrams
1 kilogram	100000 centigrams
1 stone	625000 centigrams
1 ton	90718474 centigrams
1 ounce	28350000 micrograms
1 pound	453600000 micrograms
1 milligram	1000 micrograms
1 gram	1000000 micrograms
1 kilogram	1000000000 micrograms
1 centigram	10000 micrograms
1 stone	6350000000 micrograms

24 Other Calculations

Miscellaneous Calculators

24.1 Mathematical Constants

Constant	Value
π - Archimedes Constant	3.141592653589793
e	2.7181828
Pythagoras Constant	$\sqrt{2}$
i	$\sqrt{-1}$
ϕ Golden Ratio	$\frac{1+\sqrt{5}}{2}$
Gamma γ	$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k} - Ln(n) = 0.577215664901$
Gelfonds Constant	e^{π}

24.2 Translators

24.2.1 Morse Code

Character	Morse Code Translation	Dit(Dah)	Dot Count	Dash Count
A	·	di-dah	1	1
B	· ··	dah-di-di-dit	3	1
C	· ·	dah-di-dah-dit	2	2
D	· ·	dah-di-dit	2	1
E	·	dit	1	0
F	· · ·	di-di-dah-dit	3	1
G	·	dah-dah-dit	1	2
H	· · ··	di-di-di-dit	4	0
I	··	di-dit	2	0
J	·	di-dah-dah-dah	1	3
K	·	dah-di-dah	1	2
L	· · ·	di-dah-di-dit	3	1
M		dah-dah	0	2
N	·	dah-dit	1	1
O		dah-dah-dah	0	3
P	··	di-dah-dah-dit	2	2
Q	·	dah-dah-di-dah	1	3
R	··	di-dah-dit	2	1
S	· · ·	di-di-dit	3	0
T		dah	0	1
U	· ·	di-di-dah	2	1
V	· · ·	di-di-di-dah	3	1
W	·	di-dah-dah	1	2
X	· ·	dah-di-di-dah	2	2
Y	·	dah-di-dah-dah	1	3
Z	· ·	dah-dah-di-dit	2	2
0		dah-dah-dah-dah-dah	0	5
1	·	di-dah-dah-dah-dah	1	4
2	· ·	di-di-dah-dah-dah	2	3
3	· · ·	di-di-di-dah-dah	3	2
4	· · · ·	di-di-di-di-dah	4	1
5	· · · · ·	di-di-di-di-dit	5	0
6	· · · ·	dah-di-di-di-dit	4	1

24.2.2 Phone Number

Phone Number Translation

$A, B, C \Rightarrow 2$

$D, E, F \Rightarrow 3$

$G, H, I \Rightarrow 4$

$J, K, L \Rightarrow 5$

$M, N, O \Rightarrow 6$

$P, Q, R, S \Rightarrow 7$

$T, U, V \Rightarrow 8$

$W, X, Y, Z \Rightarrow 9$

24.2.3 Affine Cipher

Affine Cipher

$E(x) = (ax + b) \bmod m$ and $D(x) = a^{-1}(x - b) \bmod m$

24.3 Bitwise Operations and Shifting

2 binary numbers a and b with digit place d

Bitwise AND a_d and b_d must both be 1

Bitwise OR either a_d or b_d must be 1

Bitwise XOR either $a_d = 1$ and $b_d = 0$ or $a_d = 0$ and $b_d = 1$

Bitwise NOT digits are reversed: $a_d = 1, \Rightarrow a_d = 0$ and $a_d = 0, \Rightarrow a_d = 1$

Bit Shift Left $a << b = a * 2^b$

Bit Shift Right $a >> b = \frac{a}{2^b}$

25 Gaming

25.1 Dice

1 Die Roll	Probability
Any Number	$\frac{1}{6}$
Even (2,4,6)	$\frac{1}{2}$
Odd (1,3,5)	$\frac{1}{2}$
Prime (1,2,3,5)	$\frac{2}{3}$
Composite (4,6)	$\frac{1}{3}$

2 Die Roll	Die Combos	Probability
Sum = 2	(1,1)	$\frac{1}{36}$
Sum = 3	(1,2), (2,1)	$\frac{1}{18}$
Sum = 4	(1,3),(3,1),(2,2)	$\frac{1}{12}$
Sum = 5	(2,3),(3,2),(1,4),(4,1)	$\frac{1}{9}$
Sum = 6	(1,5),(5,1),(2,4),(4,2),(3,3)	$\frac{5}{36}$
Sum = 7	(1,6),(6,1),(2,5),(5,2),(3,4),(4,3)	$\frac{1}{6}$
Sum = 8	(2,6),(6,2),(3,5),(5,3),(4,4)	$\frac{5}{36}$
Sum = 9	(3,6),(6,3),(4,5),(5,4)	$\frac{1}{9}$
Sum = 10	(4,6),(6,4),(5,5)	$\frac{1}{12}$
Sum = 11	(5,6),(6,5)	$\frac{1}{18}$
Sum = 12	(6,6)	$\frac{1}{36}$
Even Sum	Sum = (2,4,6,8,10)	$\frac{1}{2}$
Odd Sum	Sum = (1,3,5,7,9)	$\frac{1}{2}$
Prime	Sum = (2,3,5,7,11)	$\frac{5}{12}$
Composite	Sum = (4,6,8,9,10,12)	$\frac{7}{12}$

25.2 5 Card Poker

5 Card Poker Hand (One 52 Card Deck)	Probability
Royal Flush	$\frac{1}{649{,}740}$
Straight Flush	$\frac{3}{216{,}580}$
Four of a Kind	$\frac{1}{4{,}165}$
Full House	$\frac{6}{4{,}165}$
Flush	$\frac{1{,}277}{649{,}740}$
Straight	$\frac{5}{1{,}274}$
Three of a Kind	$\frac{88}{4{,}165}$
Two Pair	$\frac{198}{4{,}165}$
Pair	$\frac{352}{833}$

25.3 Yahtzee

Yahtzee Roll	Probability
Yahtzee	$\frac{1}{1{,}296}$
Four of a kind	$\frac{25}{1{,}296}$
Full House	$\frac{25}{648}$
Large Straight	$\frac{5}{162}$
Small Straight	$\frac{10}{81}$
Three of a kind	$\frac{25}{162}$
Chance	$\frac{205}{324}$

25.4 Lotto

Given a Lotto Drawing with p picks from n total picks, our probability is $P = \frac{p!}{\frac{n!}{(n-p)!}}$

25.5 Roulette

Roulette Bet	Probability	Expected Return on $1 bet
Red	$\frac{9}{19}$	$-0.05
Black	$\frac{9}{19}$	$-0.05
Odds	$\frac{9}{19}$	$-0.05
Evens	$\frac{9}{19}$	$-0.05
1-12	$\frac{6}{19}$	$-0.37
13-24	$\frac{6}{19}$	$-0.37
25-36	$\frac{6}{19}$	$-0.37
One Number	$\frac{1}{38}$	$-0.95
Two Numbers	$\frac{1}{19}$	$-0.89
Four Numbers	$\frac{2}{19}$	$-0.79

26 Resources and Products

I can be found using the following channels:

- Email: don@mathcelebrity.com
- Twitter: MathCelebrity

I sell the following products and services at MathCelebrity Products

- Introductory Programming Course
- Math Based Algorithms Course
- Pattern Recognition Course
- Free Traffic Secrets course. This contains the content mentioned in the "How to get 200,000 monthly visitors chapter" as well as many more tips I've learned over the years
- Personal Homework Coach
- Consulting Services

You can find these at http://www.mathcelebrity.com/products.php

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http://www.mathcelebrity.com/charity.php

http://www.mathcelebrity.com/onesecondmath.com

27 Glossary

HTML API Positive Feedback Loop PHP

Glossary

API An Application Programming Interface (API) is a particular set of rules and specifications that a software program can follow to access and make use of the services and resources provided by another particular software program that implements that API. 112

HTML Hyper Text Markup Language: A web programming language. 112

PHP Hyper Text Pre Processor: A server side programming language. 112

Positive Feedback Loop A positive feedback loop is where you do something, it works, and you do more of it which in turn feeds more momentum to your behavior. 112